Study on the bunching-antibunching nature of current noise cross correlation in ultra-small solid state entangler with ohmic dissipation based on the non-perturbative Schwinger-Keldysh scheme theory for full counting statistics

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ABSTRACT

In this thesis, we propose the the non-perturbative theory for full counting statistics (FCS) in solid state entangler (SSE) based on Nambu-Gor'kov and Schwinger-Keldysh field theoretical non-equilibrium method. Based on the theory we study the currents and cross correlation of the current noise of SSE to get the further understanding of physics of quantum entanglement. The theory can be applicable to the transport properties in the region where Coulomb blockade and Andreev reflection coexist, since tunneling processes are taken into account non-perturbatively in the presence of arbitrarily large charging effect as well as electromagnetic environment effect. Concerning the latter, we treat SSE with ohmic resistance phenomenologically in the spirit of Caldeira-Leggett theory.

The SSE considered is a coupled ultra-small double tunnel junction, structure of which consists of a common superconducting electrode/normalconducting left and right ultra-small central electrodes (islands)/normalconducting left and right drains. Each of islands are capacitively coupled with gate electrodes to control tunneling. We call the SSE double S/N/N capacitively coupled single electron transistors (*double S/N/N C-SET*). This kind of SSE is called the Cooper pair splitter (CPS) in general.

Based on the theory, we derive explicit expression for cumulant generating function (CGF) for FCS. Explicit expressions for the currents and cross correlation of current noise for the double S/N/N C-SET are obtained as the first and second cumulants from CGF, respectively. Since we are interested in extracting and controlling quantum entanglement information by the charging effect, we mainly focus our attention on the study of SSE with $U < \Delta$ (U: charging energy, Δ: energy gap of superconductor) so that sufficiently wide superconducting subgap region can be expected.

We show that, in the subgap region there are three kinds of currents due to the direct Andreev reflection (dAR), crossed Andreev reflection (cAR) and elastic cotunneling (EC). Each of the currents show Coulomb blockade related phenomena (Coulomb gaps, Coulomb staircases, and Coulomb oscillations) due to the charging effect.

Since contribution to the cross correlation of current noise $S_{\mathrm{LR}}(0)$ comes from currents due to cAR and EC in subgap region, $S_{\mathrm{LR}}(0)$ is strongly influenced by the charging effect. It is shown that $S_{\mathrm{LR}}(0)$ is always positive (bunching correlation) if two C-SETs are biased symmetrically. The bunching correlation is not genuine one (antibunching correlation) peculiar to the normal fermion flow. It is the direct consequence of the fermion flow with quantum entanglement. $S_{\mathrm{LR}}(0)$ also shows *bunching-antibunching crossover* followed by *restoration of bunching correlation* as bias voltage increases; *i.e.*, $S_{\mathrm{LR}}(0)$ becomes negative only in a narrow window of bias voltage as far as bias condition for two C-SETs is not so asymmetric enough. As the asymmetry in bias condition becomes sizable, $S_{\mathrm{LR}}(0)$ shows *bunching-antibunching crossover* only and never shows the restoration of bunching correlation.

We also show that the ohmic resistance strongly influences on $S_{\mathrm{LR}}(0)$, partly because of increase in Coulomb Gap of cAR current, and partly because of decrease in magnitude of cAR current due to the de-coherence by ohmic dissipation.

We propose the method of extraction and control of quantum entanglement information by the ferromagnetic ordering. We consider *double S/F/N C-SET*, structure of which is the one with the nonmagnetic islands of double S/N/N C-SET is replaced by ferromagnetic islands. It is shown that only cAR current (EC current) contributes to $S_{\mathrm{LR}}(0)$ if the half metal ferromagnetic islands are employed as anti-parallel (parallel) alignment.

Theoretical predictions stated above are made for the first time in this study and the results lead us to the new stage of the study on bunching-antibunching nature of current noise cross correlation in the presence of quantum entanglement.

Contents

Chapter 1

INTRODUCTION

1.1 Historical Background of the Study

Recently quantum information technology such as quantum cryptography, quantum teleportation and quantum computers have attracted much attentions. In 1935 Einstein, Podolsky and Rosen raised an objection to the quantum mechanics, so called EPR paradox [1]. In 1964 J. S. Bell found inequality for the measurements that exactly holds as far as observations are understood within the local realism. The inequality (*Bell inequality*) insists that there should exist phenomena which cannot be explained by the local realism but quantum mechanics can describe these phenomena as well as those explained by local realism. The existence of the quantum entanglement was first proved by an experiment of measuring the polarization of photons [2] through the test of Bell inequality [3]. Therefore, the essence of what once puzzled them is nowadays referred to as *quantum entanglement* [4] or *non-locality of quantum mechanics*. Although EPR paradox is not paradox but their criticism is very much important to understand quantum mechanics correctly. Because of that, the EPR paradox is often called *creative error*. Discovery of the Bell inequality is considered one of the most profound discoveries in science.

As another stream of research trend, on the other hand, physics in ultra-small systems has attracted much attention among researchers: *physics of mesoscopic systems* [5]. The study of mesoscopic systems was initiated around middle of 1980's side by side with the advance of micro-fabrication technology. Researches on the mesoscopic phenomena have been giving a big impact on not only fundamental physics but also application-oriented science. In order to understand physics in mesoscopic systems, we need much deeper understanding of basics of statistical mechanics; *reservoir, insufficient energy average, non-equilibrium, noise (fluctuations)* and so on. We have to often encounter the violation of *fluctuation-dissipation theorem*. Landauer's famous words, *" The noise is the signal."* expresses the importance on the research of the physics of mesoscopic system [6]. As for application-oriented direction, *Coulomb blockade*, which is one of the typical mesoscopic phenomena due to *charging effect*, for example, is considered a promising operation principle of *single electronics* which is expected as a new type of electronic basic devices in 21st century. Research and development of *quantum computer* which utilizes Coulomb blockade are also going on.

1.2 Present Research Status of Quantum Entanglement in Solid State Entangler

There have been so far many experimental studies on the quantum entanglement. Historically most of researches have been made in terms of photons, and some of the entanglement-based technologies are already being used although they still stay at the experimental level. Along the second research trend stated above, researches on the the quantum entanglement in ultra-small (mesoscopic) *solid state entangler (SSE)* have been extensively going on for the past ten years [7–13].

In 2009 Hofstetter et al. [7] reported an experimental research which showed synchronized Coulomb oscillation. Their experimental setup is the double tunable quantum dots coupled to a superconducting electrode and double normal electrode (Fig.1.1). They call it *Cooper pair splitter* (CPS) and insisted that they observed the quantum entanglement. However they did not reported the cross correlation of current noise.

The Cooper pair in BCS superconductivity is a spin singlet state,

$$\frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right] \ ,$$

which is nothing but spin-entangled state. The CPS utilizes possible transmission of the two electrons as a constituent of the Cooper pair to two different normalconducting paths. This kind of transmission had not been observed for a long period until report of Hofstetter *et al.*, although Andreev theoretically discussed Cooper pair dissociation into two electrons and *vice versa* at superconducting (S) and normalconducting (N)

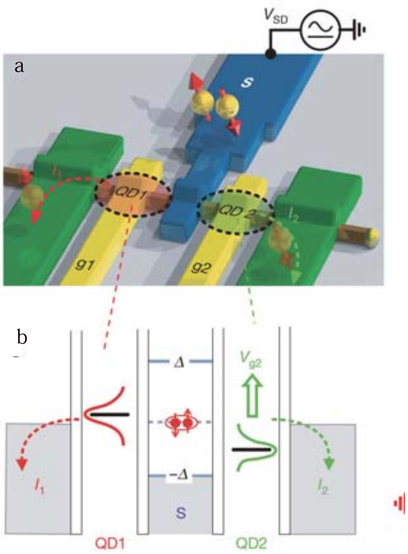

Fig. 1.1 (a) The experimental model by Hofstetter et al. An InAs nanowire is contacted by a central superconducting lead (blue) and two metallic leads (green). Two quantum dots (QD1 and QD2) form between these contacts (dotted circles). They are independently tunable by two top gates, g1 and g2. (b) The band diagram in the non-local measurement. The gate voltage of QD1 V_{g1} is fixed and the gate voltage of QD2 V_{g2} is varied. [7]

interface in the case of single S/N junction [14]. This is not the pair breaking process due to external fields, but one of the higher order equilibrium tunneling process at S/N interface. Original one is called *direct Andreev reflection* and the other called *crossed Andreev reflection*.

It is instructive to note that measurement of the cross correlation of current noise in this configuration is an analogue of measurement of interference between transmission and reflection noises in photon beam splitter. Historically this is called *Hanbury-Brown and Twiss effect* in astronomy, since they measured the apparent angular size of Sirius, claiming excellent resolution [15].

As for other experiments of CPS, studies of semiconducting nanowires [8–10], carbon nanotubes [11, 12], and graphene [13] have been made. They reported positive non-local conductance. Although there is no doubt that the charging effect is essentially important to extract and control quantum entanglement since systems are ultra-small, these experiments were all made SSE with $\Delta < U$ (Δ: the energy gap of the superconductor, U: charging energy). In this situation, only small signal can be observable. This is the reason why these experimental resorted to not current itself but sensitive non-local differential conductance tuned by gate voltage only in a narrow range of bias voltage. Nevertheless, these experimental findings still contain rather large measurement errors, so that much more careful observation should be required to get reliable signals of quantum entanglement. In the opposite case $\Delta > U$, on the other hand, we can expect much larger signals of not only non-local conductance but also currents. Furthermore, we can utilize even the bias voltage control as well as the gate voltage control over the much wider bias range. Therefore, we mainly focus our attention on the SSE with $\Delta > U$.

So far many theoretical studies have been reported on current and cross correlation of current noise or nonlocal differential conductance [16–28]. It is known that the crossed correlation of fermion flow shows negative correlation (*antibunching correlation*) [29, 30]. However it is not always true if fermion flow conveys quantum entanglement. Positive current noise cross correlation, which is normally expected for the bosonic flow and is called *bunching correlation*, has been shown theoretically for a coupled two single S/N junctions (a superconducting electrode connected with spatially separated two normalconducting electrodes) [19,22,24,25,31,32]. Among them, current noise cross correlation based on full counting statistics (FCS) scheme was studied for the structure similar to the experimental setup by Hofstetter *et al.* [24]. Although they took account of Coulomb interaction for the case $\Delta > U$, they could not show charging effect-related phenomena such as Coulomb gap, Coulomb oscillation and Coulomb staircase not only in the currents but also the current noise cross correlation. Furthermore they did not take the electromagnetic environment effect (EMEE) into consideration while EMEE should be inevitably important for the stability of Coulomb blockade [25]

Some other theoretical proposals was reported for SSE consisting of a superconductor connected with spatially separated two ferromagnetic leads [33, 34] and discussed bunching-antibunching nature. However, in this case also, the charging effect was not taken into account.

1.3 Purpose of the Study and Organization of the Dissertation

In this dissertation, we propose the non-perturbative theory for full counting statistics (FCS) in solid state entangler (SSE) based on Nambu-Gor'kov and Schwinger-

Keldysh field theoretical non-equilibrium method. Based on the theory we study the currents and cross correlation of the current noise of SSE to get the further understanding of physics of quantum entanglement. The theory can be applicable to the transport properties in the region where Coulomb blockade and Andreev reflection coexist, since tunneling processes are taken into account non-perturbatively in the presence of arbitrarily large charging effect as well as electromagnetic environment effect. Concerning the latter, we treat SSE with ohmic resistance phenomenologically in the spirit of Caldeira-Leggett theory.

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Based on the theory, we derive explicit expression for cumulant generating function (CGF) for FCS. Explicit expressions for the currents and cross correlation of current noise for the double S/N/N C-SET are obtained as the first and second cumulants from CGF, respectively. Since we are interested in extracting and controlling quantum entanglement information by the charging effect, we mainly focus our attention on the study of SSE with $U < \Delta$ (U: charging energy, Δ: energy gap of superconductor) so that sufficiently wide superconducting subgap region can be expected.

We show that, in the subgap region there are three kinds of currents due to the direct Andreev reflection, crossed Andreev reflection and elastic co-tunneling. Each of the currents show Coulomb blockade related phenomena (Coulomb gaps, Coulomb staircases and Coulomb oscillations) due to the charging effect. We also shows that the cross correlation of current noise in double S/N/N C-SET exhibits a variety of behaviors such as perfect bunching correlation, bunching-antibunching crossover, restration of bunching correlation depending on bias condition. We also discuss the effect of de-coherence caused by ohmic dissipation on these behaviors. We also discuss extraction and control method of quantum entanglement by the ferromagnetic ordering in detail.

This dissertation is organized as follows. Preliminaries are given in Chapter 2. In Chapter 3 we formulate the theory for full counting statistics for double S/N/N C-SET. Here we drive CGF and currents and current noise cross correlation from CGF and are analytically and numerically discussed in detail. In Chapter 4 we discuss the extraction and control of quantum entanglement by ferromagnetic ordering. Chapter 5 is apportioned to summary.

Chapter 2

PRELIMINARIES

2.1 Quantum Entanglement and Bell Inequality

In 1935 Einstein, Podolsky and Rosen proposed a thought experiment and raised an objection to the quantum mechanics. It is called *EPR paradox* [1]. Although it is known that EPR paradox is not a paradox, but this criticism played a very important role in right understanding of quantum mechanics. In 1964 Bell discovered so-called *Bell inequality* [3]. The inequality only holds for physical states without non-locality and tells us the essence of quantum mechanics (*non-locality of quantum mechanics* and existence of *quantum entangled states*). This "no-go theorem" is considered one of the most profound discoveries in science and opened the way to experimentally check the existence of entangled states [2].

Note that there exist natural phenomena which cannot be described by local realism and quantum mechanics specifies even such phenomena as well as those described by local realism.

2.1.1 Bell inequality (CHSH inequality)

Bell inequality can be expressed in several ways. In what follows we explain CHSH inequality which is most familiar expression for the Bell inequality [35]. Let us consider a pair of particles which have quantum spins of 1/2 ($\boldsymbol{s}_i = \frac{1}{2} \cdot \boldsymbol{\sigma}_i$, $i = 1, 2$). Components along magnetization axes $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ and $\boldsymbol{d}$ are given by $a = \boldsymbol{a} \cdot \boldsymbol{\sigma}_1$, $b = \boldsymbol{b} \cdot \boldsymbol{\sigma}_1$, $c = \boldsymbol{c} \cdot \boldsymbol{\sigma}_2$ and $d = \boldsymbol{d} \cdot \boldsymbol{\sigma}_2$. Bell introduced the hidden variable λ which cannot be observed but may affect the measurements in unknown way and its probability distribution function satisfies

$$\rho(\lambda) \geq 0 \; , \int \mathrm{d}\lambda \; \rho(\lambda) = 1. \tag{2.1}$$

Suppose measurement results for a, b, c and d, which are determined by each direction and λ, are A, B, C and D, respectively:

$$A(\boldsymbol{a}, \lambda) = \pm 1, \;\; B(\boldsymbol{b}, \lambda) = \pm 1, \;\; C(\boldsymbol{c}, \lambda) = \pm 1, \;\; D(\boldsymbol{d}, \lambda) = \pm 1. \tag{2.2}$$

The expectation value of the product of the two components is, for example, given by

$$P(\boldsymbol{a}, \boldsymbol{b}) = \int \mathrm{d}\lambda \rho(\lambda) A(\boldsymbol{a}, \lambda) B(\boldsymbol{b}, \lambda) \; . \tag{2.3}$$

In this case CHSH inequality is given by

$$|P(\boldsymbol{a}, \boldsymbol{b}) - P(\boldsymbol{a}, \boldsymbol{c}) + P(\boldsymbol{b}', \boldsymbol{b}) + P(\boldsymbol{b}', \boldsymbol{c})| \leq 2 \; , \tag{2.4}$$

which always holds as far as local hidden variable theory (local objective theory) is concerned.

2.2 Coulomb Blockade

Single electron transistor (SET) which realizes controlled single electron tunneling was theoretically proposed by Likharev [36] and has been studied extensively for the purpose of fundamental study of tunneling phenomena as well as exploration of future device with entirely new operation principle. Basic structure of SET is an ultra-small metallic double tunnel junction. Among them the SET with gate electrode capacitively coupled to the ultra-small central electrode (island) is called C-SET (*capacitively coupled single electron transistor*) [5, 36, 37]. Single electron tunneling phenomena is caused by charging energy which becomes dominant in ultra-small systems (mesoscopic systems). Under certain conditions the charging effect completely suppresses tunneling of even an electron. This is called Coulomb blockade [5].

2.2.1 Charging effect

We consider the single tunneling junction. The change of electrostatic energy ΔE by single electron tunneling is given by

$$\Delta E = \frac{(Q-e)^2}{2C} - \frac{Q^2}{2C} = \frac{e}{C}\left(\frac{e}{2} - Q\right), \tag{2.5}$$

where C, Q and e describe capacitance of tunnel junction, effective surface charge induced on the island and the charge of an electron. When $\Delta E > 0$, namely $Q < e/2$, tunneling is suppressed since energy of final state is higher than that of initial state. At low enough temperatures, even single electron tunneling is prohibited in principle. This is called charging effect and suppression of tunneling due to charging effect is called Coulomb blockade. Because of that voltage up to E_{c}/e, where

$$E_{\mathrm{c}} \equiv \frac{e^2}{2C}. \tag{2.6}$$

is the single electron charging energy, no current flows. This voltage region is called Coulomb gap.

2.2.2 Stability conditions for Coulomb blockade

In order to observe Coulomb blockade charging energy must be much larger than characteristic energies of thermal fluctuation, quantum fluctuation, and charge fluctuation.

(1) thermal fluctuation

In order to overcome the thermal fluctuation and observe the effect of charging energy, it is necessary to satisfy the condition

$$E_{\mathrm{c}} \gg k_{\mathrm{B}}T\ . \tag{2.7}$$

Therefore upper limit of temperature to utilizing Coulomb blockade is given by

$$T_0 = \frac{e^2}{2k_{\mathrm{B}}C}. \tag{2.8}$$

(2) quantum fluctuation

Characteristic energy of quantum fluctuation is considered as the energy of the level broadening due to tunneling. According to uncertainty relation, it is estimated as

$$\frac{\hbar}{R_{\mathrm{T}}C} , \tag{2.9}$$

where R_{T} is tunneling resistance given as

$$R_{\mathrm{T}}^{-1} \equiv \frac{4\pi e^2}{\hbar} |T|^2 N_0^2 , \tag{2.10}$$

where T is tunnel matrix and N_0 is density of state at Fermi energy. If E_{c} is smaller than this characteristic energy, tunneling loses its meaning. Therefore, it is necessary to satisfy the condition

$$R_{\mathrm{T}} \gg R_q \equiv \frac{\hbar}{2e^2} \sim 12.9[k\Omega] , \tag{2.11}$$

to see Coulomb blockade, where R_q is quantum resistance.

(3) charge fluctuation

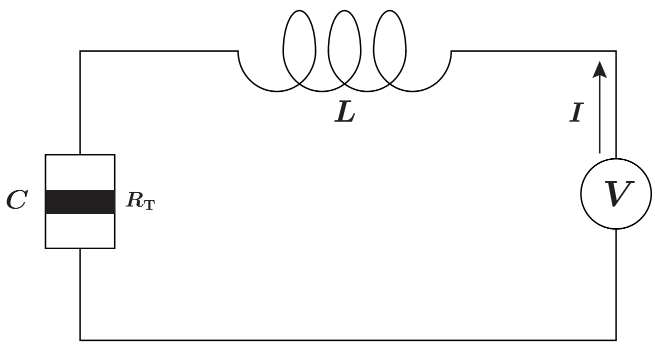


Fig. 2.1 The single electron tunnel junction which has an inductance L as external circuit. The external impedance is $Z(\omega) = \mathrm{i}\omega L$.

Single tunnel junction which has an inductance L as external circuit as shown in Fig.2.1, its eigenfrequency is $\omega_L = 1/\sqrt{LC}$, and charging Q is given by

$$Q = \mathrm{i}\sqrt{\frac{\hbar\omega_L C}{2}}(b - b^\dagger), \tag{2.12}$$

where $b^\dagger(b)$ is creation (annihilation) operator of photon. The charge fluctuation is straightforwardly calculated as

$$\langle \delta Q^2 \rangle = \frac{e^2}{2} \frac{\hbar\omega_L}{E_{\mathrm{c}}} \left\{ \frac{1}{\exp(\beta\hbar\omega_L) - 1} + \frac{1}{2} \right\}. \tag{2.13}$$

In low impedance case where $E_{\mathrm{c}} \ll \hbar\omega_L$, the charge fluctuation $\sqrt{\langle \delta Q^2 \rangle}$ becomes larger than e and Coulomb blockade can not be observed. In opposite case, $E_{\mathrm{c}} \gg \hbar\omega_L$, the charge fluctuation can be much smaller than e and Coulomb blockade is expected. Since ω_L is regarded as environmental mode, such an effect is called electromagnetic environment effect. Although impedance of the circuit considered above contains only inductance, ohmic resistance should be included in realistic case. Therefore, electromagnetic environment effect has closely related to the energy dissipation of the system.

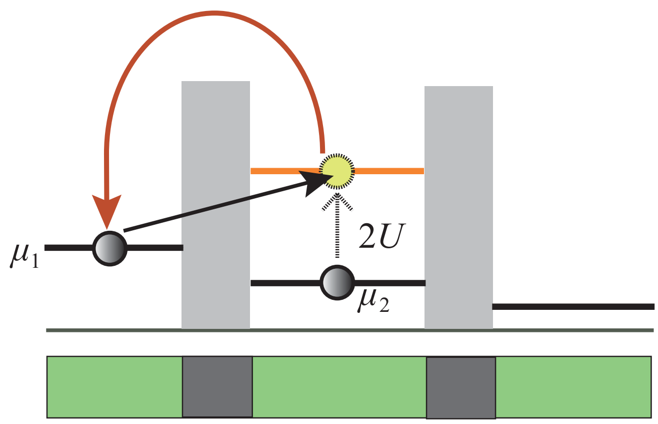


Fig. 2.2 The double tunnel junction. $U \equiv e^2/2C_\Sigma$ is the single electron charging energy in the island with $C_\Sigma \equiv C_1 + C_2$ which is capacitance of island.

In the case of double tunneling junction as shown in Fig.2.2, the alteration in stability condition of Coulomb blockade arises. This structure has a central electrode (island) which is completely separated from external circuits, so that the charging of the island is made only by electron tunneling. It means that the island charge is quantized as a direct consequence of quantization of the electron charge in unit of e. As a result the island charge never fluctuates even in the low impedance case. Therefore the Coulomb blockade can be expected if

$$\mu_1 - \mu_2 < 2U \ , \tag{2.14}$$

where

$$U \equiv e^2/2(C_1 + C_2). \tag{2.15}$$

is the single electron charging energy in the island. Note that the electromagnetic environment effect should be still taken into account even in the double junction since it affects quantitatively on the current-voltage characteristics.

2.3 Andreev Reflection

Let us consider the electron transport at the interface formed by superconducting and normalconducting electrodes (S/N interface). If bias voltage eV is larger than energy gap of the superconductor Δ, an electrons in normal conductor is able to transmit into quasi-particle energy level of the superconductor. Reverse process is Cooper pair breaking. An electron which is one of the constituent of a Cooper pair is excited to the quasi-particle level and another constituent electron transmits into the normalconductor. This is so-called *quasi-particle tunneling*.

An interesting situation arises when one considers (at zero temperature) the flow of the electric current, driven by an infinitesimally small bias, $\delta\mu = e\delta V$, through a clean S/N interface. In the normal conductor it is carried by quasi-particle, while in a superconductor there are no quasi-particles with energies less than Δ, and the current is carried by the condensate of Cooper pairs. Therefore, some conversion mechanism should be required.

This mechanism was first pointed out by Andreev and is called *Andreev reflection* [14]. The process is schematically shown in Fig.2.3. An electron incident from the normalconductor with energy E above the Fermi level E_{F} cannot transmit into the superconductor. Instead, it picks up another electron with the same energy, almost the same momentum and opposite spin, and forms a Cooper pair and joins the condensate in the superconductor. Thus this process carries the net momentum and the electric current of *two* electrons across the S/N interface. This leaves a hole on the normal side. The Bloch velocity of hole, $v_{\boldsymbol{k}} = \nabla_{\boldsymbol{k}}\epsilon_{\boldsymbol{k}}$ is *opposite* to its momentum, so the hole moves away from the interface and retraces the path of the incoming electron. The whole process can, therefore, be considered as an *reflection* of an electron. The reverse process, which is the Andreev reflection of a hole, realizes the hole-electron conversion. Although we do not go into the details of this process, Andreev reflection can be well described by the Bogoliubov-de Gennes equation method or tunneling Hamiltonian method. In this dissertation study we take the latter method. In this approach, Andreev reflection process can be described as higher order tunneling process than the quasi-particle tunneling: the lowest order of Andreev mechanism is proportional to $|T|^4$, while quasi-particle mechanism $|T|^2$ in terms of tunneling matrix elements. Therefore, as much as the S/N interface is clean (S/N interface with higher transmission rate), the Andreev process becomes more dominant. Remembering Coulomb blockade becomes conspicuous for the low transmission interface, we should realize that non-perturbative treatment of tunneling is indispensable to the discussions in the situation where Coulomb blockade and Andreev process coexist.

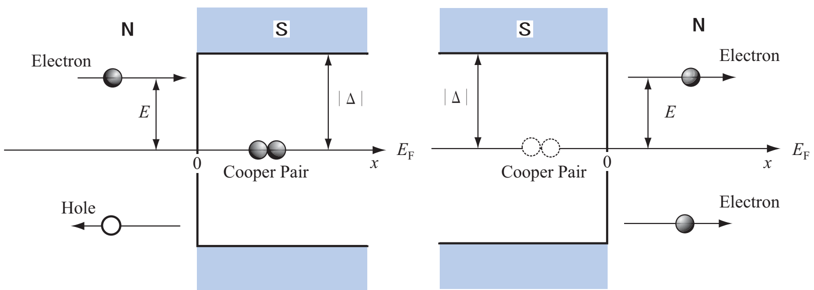


Fig. 2.3 Andreev reflection

2.4 Quantum Statistical Method for Equilibrium Systems

2.4.1 Time evolution of density matrix operator

Let us consider the system described by the following Hamiltonian:

$$H = H_0 + H', \tag{2.16}$$

where H_0 and H' are, respectively, unperturbed and perturbative parts of the Hamiltonian of the system. Let us consider the operator (observable) O. Expressions of the observable in the interaction and the Heisenberg pictures are as follows:

$$O_{\rm I}(t) \equiv {\rm e}^{{\rm i}\frac{H_0}{\hbar}t} O {\rm e}^{-{\rm i}\frac{H_0}{\hbar}t} , \tag{2.17}$$

$$O_{\rm H}(t) \equiv {\rm e}^{{\rm i}\frac{H}{\hbar}t} O {\rm e}^{-{\rm i}\frac{H}{\hbar}t} . \tag{2.18}$$

Expressing $O_{\rm H}(t)$ in terms of interaction picture, we have

$$\begin{aligned} O_{\rm H}(t) &= {\rm e}^{{\rm i}\frac{H}{\hbar}t} {\rm e}^{-{\rm i}\frac{H_0}{\hbar}t} O_{\rm I}(t) {\rm e}^{{\rm i}\frac{H_0}{\hbar}t} {\rm e}^{-{\rm i}\frac{H}{\hbar}t} \\ &= U(0,t) O_{\rm I}(t) U(t,0), \end{aligned} \tag{2.19}$$

where $U(t, t_0)$ is the time evolution operator for the state vectors in the interaction picture and is defined by

$$U(t,t_0) = {\rm e}^{{\rm i}\frac{H_0}{\hbar}t} {\rm e}^{-{\rm i}\frac{H}{\hbar}(t-t_0)} {\rm e}^{-{\rm i}\frac{H_0}{\hbar}t_0}. \tag{2.20}$$

Noting that the time evolution operator $U(t, t_0)$ satisfies

$$ {\rm i}\hbar \frac{\partial}{\partial t} U(t,t_0) = H'_{\rm I}(t) U(t,t_0) , \tag{2.21}$$

where $H'_{\rm I}$ is the interaction picture of the H', we arrive at another expression of $U(t, t_0)$:

$$U(t,t_0) = T \exp\left[-\frac{\rm i}{\hbar} \int_{t_0}^{t} {\rm d}t' H'_{\rm I}(t') \right] , \tag{2.22}$$

where T is the Wick's chronological operator.

The density matrix operator ρ is defined by

$$\rho \equiv \frac{{\rm e}^{-\beta H}}{{\rm Tre}^{-\beta H}} , \tag{2.23}$$

and it evolves in time according to the von Neumann equation:

$$ {\rm i}\hbar \frac{\partial \rho}{\partial t} = [\, H, \rho \,] . \tag{2.24}$$

The density matrix operator in the interaction picture is defined by

$$\rho_{\rm I}(t) \equiv {\rm e}^{{\rm i}\frac{H_0}{\hbar}t} \rho {\rm e}^{-{\rm i}\frac{H_0}{\hbar}t} , \tag{2.25}$$

and its time evolution is given as

$$ {\rm i}\hbar \frac{\partial}{\partial t} \rho_{\rm I}(t) = [\, H'_{\rm I}(t) \, , \rho_{\rm I}(t) \,]. \tag{2.26}$$

After some manipulations, the solution of Eq.(2.26) satisfies the following relation:

$$\rho_{\rm I}(t) = U(t,t') \rho_{\rm I}(t') U(t',t) . \tag{2.27}$$

2.4.2 Quantum statistical average of observables

Let us consider the adiabatic switching on of perturbation as usual. Starting point is the following Hamiltonian:

$$H = H_0 + H' \mathrm{e}^{-\delta|t|}, \tag{2.28}$$

where δ is an infinitesimal positive quantity. At very large times, both in the past and in the future, the Hamiltonian is reduced to H_0, which is assumed to be a soluble problem. The density matrix operator is then

$$\rho(t = -\infty) \equiv \rho_0 \equiv \frac{\mathrm{e}^{-\beta H_0}}{\mathrm{Tre}^{-\beta H_0}} \ . \tag{2.29}$$

As t increases from $-\infty$, interaction turned on, and state vector in the interaction picture develops in time, all the way to the time $t = 0$, when the interaction is at full strength, H'. Then the expectation value of observable $O(t)$ can be represented in

$$\begin{aligned}\langle O_{\mathrm{H}}(t) \rangle &= \mathrm{Tr}[\rho \, O_{\mathrm{H}}(t)] = \mathrm{Tr}[\rho_{\mathrm{I}}(0) O_{\mathrm{H}}(t)] \\ &= \mathrm{Tr}[U(0, -\infty)\rho_{\mathrm{I}}(-\infty)U(-\infty, 0)U(0, t)O_{\mathrm{I}}(t)U(t, 0)] \\ &= \mathrm{Tr}[\rho_0 U(-\infty, t)O_{\mathrm{I}}(t)U(t, -\infty)] \\ &\equiv \langle U(-\infty, t)O_{\mathrm{I}}(t)U(t, -\infty)\rangle_0 \ . \end{aligned} \tag{2.30}$$

This is the general formula to calculate statistical average of observables.

2.5 Quantum Statistical Method for Non-Equilibrium Systems

In this section, we review Keldysh method [38, 39]. In ultra-small systems such as mesoscopic systems, even small external field easily drives such systems into highly non-equilibrium states. Taking account of non-linear response of the system caused by external field is also indispensable to reliable description of the non-equilibrium system. In Keldysh method we can also utilize various exact relations which enable us to treat the problem non-perturbatively.

2.5.1 Closed time contour (Keldysh contour)

The Gell-Mann and Low theorem states that this prescription generates the eigenstate of $H_0 + H'$ that develops adiabatically from eigenstate of H_0. Furthermore, since H becomes H_0 at $t = \pm\infty$ in this prescription, as far as the system remains equilibrium state, the difference in state vectors at $t = \pm\infty$ is only the phase factor. Consequently it is not necessary for us to pay the special care about the states at $t = \pm\infty$ when we consider quantum statistical average of observables. However, if H' drives the system to non equilibrium state, there is no guarantee for the validity of the Gell-Mann and Low theorem. Breakdown of equilibrium may cause the essential difference between state vectors at $t = \pm\infty$, so that we have to go back to the initial state that we only know what it is like, whenever we consider quantum statistical average of observable.

Utilizing relation

$$U(t_2, t_1) = U(t_2, t')U(t', t_1), \tag{2.31}$$

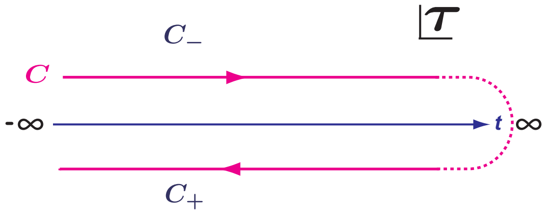


Fig. 2.4 Keldysh contour

Eq.(2.30) should be treated as

$$\langle O_{\rm H}(t)\rangle = \langle U(-\infty,\infty)U(\infty,t)O_{\rm I}(t)U(t,-\infty)\rangle_0. \tag{2.32}$$

The operator $U(-\infty,\infty)$ should be considered time evolution operator which go back in time along different time contour C_+ from the time contour C_- for $U(\infty,-\infty)$ as shown in Fig.2.4, and is defined by

$$U(-\infty,\infty) = \tilde{T}\exp\left[\frac{\rm i}{\hbar}\int_{-\infty}^{\infty}{\rm d}t' H_{\rm I}'(t')\right], \tag{2.33}$$

where $\tilde{T}$ is the anti-chronological operator which set in order oppositely from how T does. Therefore, Eq.(2.32) can be expressed as

$$\begin{aligned}\langle O_{\rm H}(t)\rangle &= \left\langle \tilde{T}\exp\left[-\frac{\rm i}{\hbar}\int_{\infty}^{-\infty}{\rm d}t' H_{\rm I}'(t')\right] T\left\{\exp\left[-\frac{\rm i}{\hbar}\int_{-\infty}^{\infty}{\rm d}t' H_{\rm I}'(t')\right]O_{\rm I}(t)\right\}\right\rangle_0 \\ &\equiv \left\langle T_{\rm C}\left\{\exp\left[-\frac{\rm i}{\hbar}\int_C {\rm d}\tau H_{\rm I}'(\tau)\right]O_{\rm I}(t)\right\}\right\rangle_0,\end{aligned} \tag{2.34}$$

where $C = C_- + C_+$ and $T_{\rm C}$ are the closed time contour as shown in Fig.2.4 and the chronological operator along C, respectively.

2.5.2 The definition of Keldysh Green's function

The Green's function according to the time path in Fig.2.4 is defined by

$$\begin{aligned}G(\boldsymbol{k},\boldsymbol{k}';t,t') &\equiv -{\rm i}\langle T_{\rm C}\, c_{\boldsymbol{k}\sigma{\rm H}}(t)c^\dagger_{\boldsymbol{k}'\sigma{\rm H}}(t')\rangle \\ &= -{\rm i}\langle T_{\rm C}U_{\rm C}c_{\boldsymbol{k}\sigma}(t)c^\dagger_{\boldsymbol{k}'\sigma}(t')\rangle_0\end{aligned} \tag{2.35}$$

called Keldysh Green's function, where $c_{\boldsymbol{k}\sigma{\rm H}}$ ($c_{\boldsymbol{k}\sigma}$) and $c^\dagger_{\boldsymbol{k}'\sigma{\rm H}}$ ($c^\dagger_{\boldsymbol{k}'\sigma}$) are annihilation and creation operators of electron in Heisenberg (interaction) picture. Furthermore $U_{\rm C}$ is time evolution operator along time contour C and is defined by

$$U_{\rm C} \equiv T_{\rm C}\exp\left[-\frac{\rm i}{\hbar}\int_C {\rm d}\tau H_{\rm I}'(\tau)\right]. \tag{2.36}$$

Keldysh Green's function defined in Eq.(2.35) satisfies the Dyson equation given by

$$G(t,t') = g(t,t') + \int_C\int_C {\rm d}\tau_1{\rm d}\tau_2 g(t,\tau_1)\,\frac{1}{\hbar}\Sigma(\tau_1,\tau_2)G(t_2,t'), \tag{2.37}$$

where the free electron Green's function g is given by

$$g(\boldsymbol{k};t,t') = -\mathrm{i}\langle T_C\, c_{\boldsymbol{k}\sigma}(t)c^\dagger_{\boldsymbol{k}\sigma}(t')\rangle_0. \tag{2.38}$$

Keldysh Green's function consists of four components depending on which time branch t and t' belong to:

$$G(\boldsymbol{k},\boldsymbol{k}';t,t') = \begin{cases} G^{--}(\boldsymbol{k},\boldsymbol{k}';t,t') & t,t' \in C_- \\ G^{+-}(\boldsymbol{k},\boldsymbol{k}';t,t') & t \in C_+, t' \in C_- \\ G^{-+}(\boldsymbol{k},\boldsymbol{k}';t,t') & t \in C_-, t' \in C_+ \\ G^{++}(\boldsymbol{k},\boldsymbol{k}';t,t') & t,t' \in C_+ \end{cases} . \tag{2.39}$$

Expressions for these four Green's function are as follows:

$$\begin{aligned} G^{--}(\boldsymbol{k},\boldsymbol{k}';t,t') &= -\mathrm{i}\langle T\, c_{\boldsymbol{k}\sigma\mathrm{H}}(t)c^\dagger_{\boldsymbol{k}'\sigma\mathrm{H}}(t')\rangle \\ &= -\mathrm{i}\theta(t-t')\langle c_{\boldsymbol{k}\sigma\mathrm{H}}(t)c^\dagger_{\boldsymbol{k}'\sigma\mathrm{H}}(t')\rangle + \mathrm{i}\theta(t'-t)\langle c^\dagger_{\boldsymbol{k}'\sigma\mathrm{H}}(t')c_{\boldsymbol{k}\sigma\mathrm{H}}(t)\rangle\ , \end{aligned} \tag{2.40}$$

$$G^{+-}(\boldsymbol{k},\boldsymbol{k}';t,t') = -\mathrm{i}\langle c_{\boldsymbol{k}\sigma\mathrm{H}}(t)c^\dagger_{\boldsymbol{k}'\sigma\mathrm{H}}(t')\rangle\ , \tag{2.41}$$

$$G^{-+}(\boldsymbol{k},\boldsymbol{k}';t,t') = \mathrm{i}\langle c^\dagger_{\boldsymbol{k}'\sigma\mathrm{H}}(t')c_{\boldsymbol{k}\sigma\mathrm{H}}(t)\rangle\ , \tag{2.42}$$

$$\begin{aligned} G^{++}(\boldsymbol{k},\boldsymbol{k}';t,t') &= -\mathrm{i}\langle \tilde{T}\, c_{\boldsymbol{k}\sigma\mathrm{H}}(t)c^\dagger_{\boldsymbol{k}'\sigma\mathrm{H}}(t')\rangle \\ &= -\mathrm{i}\theta(t'-t)\langle c_{\boldsymbol{k}\sigma\mathrm{H}}(t)c^\dagger_{\boldsymbol{k}'\sigma\mathrm{H}}(t')\rangle + \mathrm{i}\theta(t-t')\langle c^\dagger_{\boldsymbol{k}'\sigma\mathrm{H}}(t')c_{\boldsymbol{k}\sigma\mathrm{H}}(t)\rangle\ , \end{aligned} \tag{2.43}$$

G^{--} is also written as G_c (time-ordered Green's function), G^{+-} as $G^>$ (the greater function), G^{-+} as $G^<$ (the lesser function), and G^{++} as $G_{\bar{c}}$ (anti time-ordered Green's function). Thus Keldysh Green's function and its self-energy can be expressed as 2×2 matrix in

$$\check{G} = \begin{pmatrix} G^{--} & G^{-+} \\ G^{+-} & G^{++} \end{pmatrix},\quad \check{\Sigma} = \begin{pmatrix} \Sigma^{--} & \Sigma^{-+} \\ \Sigma^{+-} & \Sigma^{++} \end{pmatrix}. \tag{2.44}$$

Such a matrix space is called the Keldysh space. The Fourier transformation is defined as usual by

$$G(\omega) = \int_{-\infty}^{\infty} \mathrm{d}\omega\ \mathrm{e}^{\mathrm{i}\omega(t-t')}G(t,t'), \tag{2.45}$$

resulting in

$$\check{G}(\omega) = \check{g}(\omega) + \check{g}(\omega)\ \frac{1}{\hbar}\ \check{\Sigma}(\omega)\ \check{G}(\omega), \tag{2.46}$$

Note that four components are not independent of each other (only three degree of freedom), since there is a relation

$$G^{--} + G^{++} = G^{-+} + G^{+-}. \tag{2.47}$$

Therefore so-called Keldysh rotation is sometimes employed. Using unitary matrix

$$\check{L} = \frac{1}{\sqrt{2}}\,(\sigma_0 - \mathrm{i}\sigma_2) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\ , \tag{2.48}$$

we have the following expressions:

$$\underline{\check{G}} = \check{L}\check{\sigma}_3\check{G}\check{L}^\dagger = \begin{pmatrix} G^{\mathrm{R}} & G^{\mathrm{K}} \\ 0 & G^{\mathrm{A}} \end{pmatrix}\ , \tag{2.49}$$

$$\check{\underline{\Sigma}} = \check{L}\check{\Sigma}\check{\sigma}_3\check{L}^\dagger = \begin{pmatrix} \Sigma^{\mathrm{R}} & \Sigma^{\mathrm{K}} \\ 0 & \Sigma^{\mathrm{A}} \end{pmatrix} , \tag{2.50}$$

where G^{R}, G^{A} and G^{K} are called retarded, advanced and Keldysh (or kinetic) components of Keldysh Green's function. Definitions of these components are given as

$$\begin{aligned} G^{\mathrm{R}}(\boldsymbol{k},\boldsymbol{k}';t,t') &= -\mathrm{i}\theta(t-t')\langle[c_{\boldsymbol{k}\sigma\mathrm{H}}(t), c^\dagger_{\boldsymbol{k}'\sigma\mathrm{H}}(t')]_+\rangle \\ &= \theta(t-t')\{G^{+-}(\boldsymbol{k},\boldsymbol{k}';t,t') - G^{-+}(\boldsymbol{k},\boldsymbol{k}';t,t')\} , \end{aligned} \tag{2.51}$$

$$\begin{aligned} G^{\mathrm{A}}(\boldsymbol{k},\boldsymbol{k}';t,t') &= \mathrm{i}\theta(t'-t)\langle[c_{\boldsymbol{k}\sigma\mathrm{H}}(t), c^\dagger_{\boldsymbol{k}'\sigma\mathrm{H}}(t')]_+\rangle \\ &= \theta(t'-t)\{G^{-+}(\boldsymbol{k},\boldsymbol{k}';t,t') - G^{+-}(\boldsymbol{k},\boldsymbol{k};t,t')\} , \end{aligned} \tag{2.52}$$

$$\begin{aligned} G^{\mathrm{K}}(\boldsymbol{k},\boldsymbol{k}';t,t') &= -\mathrm{i}\langle[c_{\boldsymbol{k}\sigma\mathrm{H}}(t), c^\dagger_{\boldsymbol{k}'\sigma\mathrm{H}}(t')]_-\rangle \\ &= G^{+-}(\boldsymbol{k},\boldsymbol{k}';t,t') + G^{-+}(\boldsymbol{k},\boldsymbol{k}';t,t') , \end{aligned} \tag{2.53}$$

where $[A,B]_\pm = AB \pm BA$.

Finally let us summarize the various important relations between components of Keldysh Green's functions and self-energies:

$$G^{\mathrm{R}} = G^{--} - G^{-+} = G^{+-} - G^{++} , \tag{2.54}$$
$$G^{\mathrm{A}} = G^{--} - G^{+-} = G^{-+} - G^{++} , \tag{2.55}$$
$$G^{\mathrm{K}} = G^{--} + G^{++} = G^{-+} + G^{+-} , \tag{2.56}$$

$$\Sigma^{\mathrm{R}} = \Sigma^{--} + \Sigma^{-+} = -(\Sigma^{+-} + \Sigma^{++}) , \tag{2.57}$$
$$\Sigma^{\mathrm{A}} = \Sigma^{--} + \Sigma^{+-} = -(\Sigma^{-+} + \Sigma^{++}) , \tag{2.58}$$
$$\Sigma^{\mathrm{K}} = \Sigma^{--} + \Sigma^{++} = -(\Sigma^{-+} + \Sigma^{+-}) , \tag{2.59}$$

$$G^{-+} =(1 + G^{\mathrm{R}}\Sigma^{\mathrm{R}})g^{-+}(1 + \Sigma^{\mathrm{A}}G^{\mathrm{A}}) + G^{\mathrm{R}}\Sigma^{-+}G^{\mathrm{A}} , \tag{2.60}$$
$$G^{+-} =(1 + G^{\mathrm{R}}\Sigma^{\mathrm{R}})g^{+-}(1 + \Sigma^{\mathrm{A}}G^{\mathrm{A}}) + G^{\mathrm{R}}\Sigma^{+-}G^{\mathrm{A}} , \tag{2.61}$$
$$G^{\mathrm{K}} =(1 + G^{\mathrm{R}}\Sigma^{\mathrm{R}})g^{\mathrm{K}}(1 + \Sigma^{\mathrm{A}}G^{\mathrm{K}}) - G^{\mathrm{R}}\Sigma^{\mathrm{K}}G^{\mathrm{A}} . \tag{2.62}$$

2.6 Full Counting Statistics

The counting statistics (CS) is originally one of the methods often adopted in the field of optics and it means that counting up the number of target physical quantity in limited time and watching the statistics with respect to the number. Recently the idea of CS has been generalized and applied to the transport properties of the electrons in ultra-small (mesoscopic) systems where the stationarity is not well guaranteed due to large fluctuations [40]. If the counting-up can be made perfectly, we can, in principle, deduce the probability distribution function with respect to number. With this meaning, the method is called full counting statistics (FCS). Here we briefly explain basics of FCS.

2.6.1 Overview of FCS

We consider the probability distribution function $P_{t_0}(N)$, where N is the number of electrons transferred in a given time interval t_0. In FCS scheme, the characteristic

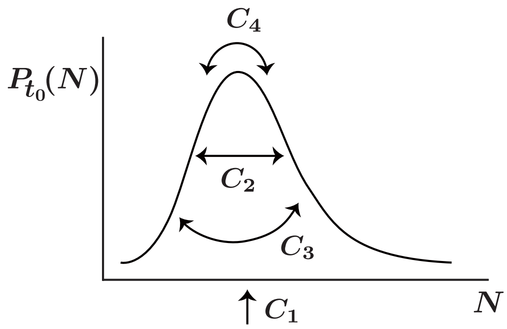


Fig. 2.5 The probability distribution as a function Number of electron N.

function (moment generating function) $Z_{t_0}(\chi)$ is defined as

$$Z_{t_0}(\chi) = \sum_{N=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\chi N} P_{t_0}(N) = \mathrm{e}^{\Omega_{t_0}(\chi)} , \tag{2.63}$$

where χ is called counting field and $\Omega_{t_0}(\chi)$ is the cumulant generating function (CGF) with finite time interval. It is assumed that the time interval t_0 is well larger than relaxation time of the system. The k-th cumulant can be obtained from the k-th derivative of CGF

$$C_k = \left. \frac{\partial^k \Omega_{t_0}(\chi)}{\partial(\mathrm{i}\chi)^k} \right|_{\chi=0} = \langle\langle N^k \rangle\rangle_{t_0}. \tag{2.64}$$

The first and second cumulants are, for example, given as

$$C_1 = \left. \frac{\partial \Omega_{t_0}(\chi)}{\partial(\mathrm{i}\chi)} \right|_{\chi=0} = \langle N \rangle_{t_0} , \tag{2.65}$$

$$C_2 = \left. \frac{\partial^2 \Omega_{t_0}(\chi)}{\partial(\mathrm{i}\chi)^2} \right|_{\chi=0} = \langle N^2 \rangle_{t_0} - \langle N \rangle_{t_0}^2 , \tag{2.66}$$

and are the mean and the variance of N, respectively. The third and forth cumulants characterize the skewness and the sharpness of the probability distribution function, respectively.

2.6.2 Introduction of counting field for double tunnel junction

Let us consider the double tunnel junction depicted in Fig.2.6. Grand canonical Hamiltonian of the system is given as

$$K = \sum_{\nu=A,B,D} K_\nu^{(0)} + \sum_{\nu=A,B} H_{\mathrm{T}}^{(\nu)} = K_0 + H_{\mathrm{T}} , \tag{2.67}$$

where $K_\nu^{(0)} = H_\nu^{(0)} - \mu_\nu N_\nu$. The first three terms describe Hamiltonians of the electrodes A, D, and B in the absence of tunneling and the second two terms describe tunnelings between electrodes. Constructing theory for FCS, the first important step is the introduction of counting field.

Let us first consider the measurement of change in N_{A} caused by $H_{\mathrm{T}}^{(\mathrm{A})}$. Measurement protocol consists of the following steps:

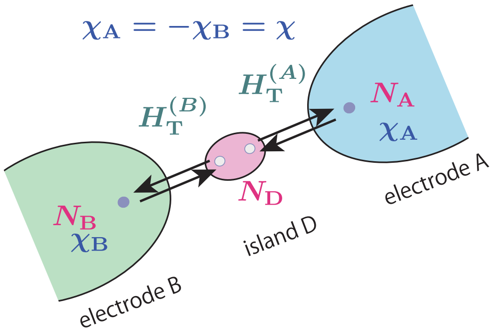


Fig. 2.6 Introduction of counting field χ for double tunnel junction.

(1) initial observation of $N_{\rm A}$ and $N_{\rm B}$ at time $t_i = -T_0/2$

State Vector : $|\psi_{\rm total}\rangle^{(i)} = |\psi_{\rm A}\rangle^{(i)} \otimes |\psi_{\rm D}\rangle^{(i)} \otimes |\psi_{\rm B}\rangle^{(i)}$

Density Matrix Operator : $\rho_{\rm G}^{(i)} = {\rm e}^{-\beta K_{\rm A}^{(0)}} \otimes {\rm e}^{-\beta K_{\rm D}^{(0)}} \otimes {\rm e}^{-\beta K_{\rm B}^{(0)}}$

$[\ K_\nu = H_\nu - \mu_\nu N_\nu \quad (\nu = A, D, B)\ ]$

results $\Rightarrow N_A^{(i)}$ and $N_B^{(i)}$

(2) time evolution T_0

(3) final observation of $N_{\rm A}$ and $N_{\rm B}$ at time $t_f = T_0/2$

State Vector : $|\psi_{\rm total}\rangle^{(f)} = |\psi_{\rm A}\rangle^{(f)} \otimes |\psi_{\rm D}\rangle^{(f)} \otimes |\psi_{\rm B}\rangle^{(f)}$

Density Matrix Operator : unknown

results $\Rightarrow N_A^{(f)}$ and $N_B^{(f)}$

A set of observation process stated above should be repeated again and again $\cdots$. After all observations we try to treat observation results statistically.

Let us formulate this protocol theoretically. Since particle number tunneling into electrode A in time interval $T_0 = t_f - t_i$ is $N_{\rm A}^{(f)} - N_{\rm A}^{(i)} (\equiv \Delta N_{\rm A})$, probability for this event is expressed as

$$P_{fi}(\Delta N_{\rm A}) \propto |^{(f)}\langle\psi_{\rm total}|U_{\rm A}(T_0)|\psi_{\rm total}\rangle^{(i)}|^2 \delta(\Delta N_{\rm A} - (N_{\rm A}^{(f)} - N_{\rm A}^{(i)}))\ , \tag{2.68}$$

where $U^{({\rm A})}(T_0)$ is the time evolution operator determined by tunnel Hamiltonian $H_{\rm T}^{({\rm A})}$. Probability distribution function $P(\Delta N_{\rm A})$ is then obtained as

$$P(\Delta N_{\rm A}) = \sum_{if} P_{fi}(\Delta N_{\rm A})\ ^{(i)}\langle\psi_{\rm total}|\rho_{\rm G}^{(i)}|\psi_{\rm total}\rangle^{(i)}\ . \tag{2.69}$$

Noting that

$$\delta\left(\Delta N_{\rm A} - (N_{\rm A}^{(f)} - N_{\rm A}^{(i)})\right) = \int_{-\infty}^{\infty} \frac{{\rm d}\chi_{\rm A}}{2\pi} {\rm e}^{-{\rm i}\chi_{\rm A}(\Delta N_{\rm A} - (N_{\rm A}^{(f)} - N_{\rm A}^{(i)}))}\ ,$$

we have

$$P(\Delta N_{\rm A}) = \int_{-\infty}^{\infty} \frac{{\rm d}\chi_{\rm A}}{2\pi} {\rm e}^{-{\rm i}\chi_{\rm A}\Delta N_{\rm A}} Z_{\rm A}(\chi_{\rm A})\ , \tag{2.70}$$

where

$$Z_{\mathrm{A}}(\chi_{\mathrm{A}};T_0) = \mathrm{Tr}\left\{\rho_{\mathrm{G}}^{(i)} \exp\left(\frac{\mathrm{i}}{\hbar}K_{\mathrm{A}+}(\chi_{\mathrm{A}})T_0\right)\exp\left(-\frac{\mathrm{i}}{\hbar}K_{\mathrm{A}-}(\chi_{\mathrm{A}})T_0\right)\right\} , \quad (2.71)$$

$$= \mathrm{Tr}\left\{\rho_{\mathrm{G}}^{(i)} U_{\mathrm{A},+}\left(\chi_{\mathrm{A}}; -\frac{T_0}{2}, \frac{T_0}{2}\right) \cdot U_{\mathrm{A},-}\left(\chi_{\mathrm{A}}; \frac{T_0}{2}, -\frac{T_0}{2}\right)\right\} , \quad (2.72)$$

$$\equiv \exp\left\{\Omega_{\mathrm{A}}(\chi_{\mathrm{A}};T_0)\right\} , \quad (2.73)$$

with

$$U_{\mathrm{A},\mp}\left(\pm\frac{T_0}{2}, \mp\frac{T_0}{2}\right) \equiv T_{C_\mp} \exp\left\{-\frac{\mathrm{i}}{\hbar}\int_{\mp T_0/2}^{\pm T_0/2} \mathrm{d}t\; H_{\mathrm{T},\mp}^{(\mathrm{A})}(\chi_{\mathrm{A}};t)\right\} , \quad (2.74)$$

$$K_{\mathrm{A},\mp}(\chi_{\mathrm{A}}) \equiv \mathrm{e}^{\pm\frac{\mathrm{i}}{2}\chi_{\mathrm{A}}N_{\mathrm{A}}} K_{\mathrm{A}} \mathrm{e}^{\mp\frac{\mathrm{i}}{2}\chi_{\mathrm{A}}N_{\mathrm{A}}} \quad (2.75)$$

$$= K_{\mathrm{A}}^{(0)} + H_{\mathrm{T},\mp}^{(A)}(\chi_{\mathrm{A}}) . \quad (2.76)$$

Here notations $T_{C-}(T_{C+})$ are nothing but $T(\tilde{T})$ in Sec.2.5.2. Deriving Eq.(2.71) from Eq.(2.69), we generalized time evolution in the sprit of Keldysh method so that we can apply the formalism to the non-equilibrium situation.

Now let us consider the measurement of change in N_{B} caused by $H_{\mathrm{T}}^{(\mathrm{B})}$. Although we do not write the corresponding expressions in this case, it is obvious that the discussion can be made exactly the same way for the change in N_{A}. What is important is that we should realize the changes in N_{A} and N_{B} are not independent, but $\Delta N_{\mathrm{B}} = -\Delta N_{\mathrm{A}}$ as far as the number conservation is retained. In other word, we should set the following conditions with respect to counting fields as a necessary condition:

$$\chi_{\mathrm{B}} = -\chi_{\mathrm{A}} . \quad (2.77)$$

Note that only Eq.(2.77) does not bring us the current of the system. In order to get it we have to further impose the current continuity condition for the determination of optimum chemical potentials of the central electrode D.

Finally let us summarize the prescription for introducing counting field for FCS:

STEP 1 Introducing time dependent counting field along Keldysh contour defined by

$$\chi_\nu(\tau) \equiv \begin{cases} \chi_\nu(t_-) = \chi_\nu & \text{for } \tau = t_- \ (\tau \text{ on } C_-) \\ \chi_\nu(t_+) = -\chi_\nu & \text{for } \tau = t_+ \ (\tau \text{ on } C_+) \end{cases} , \quad (2.78)$$

for

$$\theta\left(t_\mp \pm \frac{T_0}{2}\right)\theta\left(-t_\mp \pm \frac{T_0}{2}\right) , \quad (2.79)$$

perform the following unitary transformation:

$$K_C(\{\chi_\nu(\tau)\};\tau) \equiv \exp\left(\frac{\mathrm{i}}{2}\sum_{\nu=A,B}\chi_\nu(\tau)N_\nu(\tau)\right) \cdot K(\tau)$$

$$\times \exp\left(-\frac{\mathrm{i}}{2}\sum_{\nu=A,B}\chi_\nu(\tau)N_\nu(\tau)\right) \quad (2.80)$$

$$= \sum_{\nu=A,B}\left[K_\nu^{(0)}(\tau) + H_{\mathrm{T}}^{(\nu)}(\chi_\nu(\tau);\tau)\right] = K_0(\tau) + H_{\mathrm{T}}(\{\chi_\nu(\tau)\};\tau) , \quad (2.81)$$

STEP 2 Impose the boundary condition on counting fields:

$$\chi_{\mathrm{A}}(\tau) = -\chi_{\mathrm{B}}(\tau) \ , \tag{2.82}$$

as necessary condition for current continuity. As a result, only one counting field $\chi(\tau)$ is introduced.

STEP 3 Cumulant generating function (CGF) of the system is then given as

$$\Omega(\chi) = \lim_{T_0\to\infty} \frac{1}{T_0} \log \left\langle \ T_C \ \exp\left\{ -\frac{\mathrm{i}}{\hbar} \int_C \mathrm{d}\tau \ H_{\mathrm{T},C}(\chi(\tau);\tau) \right\} \ \right\rangle_{0,T_0} , \tag{2.83}$$

where $\langle \cdots \rangle_{0,T_0}$ denotes average with respect to K_0 along Keldysh contour with finite time interval $[-T_0/2, T_0/2]$ and

$$H_{\mathrm{T},C}(\chi(\tau);\tau) \equiv \begin{cases} H_{\mathrm{T},-}(\chi;t) & \text{for } t = t_- \ (\tau \text{ on } C_-) \\ H_{\mathrm{T},+}(\chi;t) & \text{for } t = t_+ \ (\tau \text{ on } C_+) \end{cases} .$$

Note that

$$H_{\mathrm{T},+}(\chi;t) = H_{\mathrm{T},-}(-\chi;t) \ , \tag{2.84}$$

and $\langle \ \cdots \ \rangle_{0,T_0} \neq \langle \ \cdots \ \rangle_0$ as far as T_0 is finite.

Chapter 3

FULL COUNTING STATISTICS IN DOUBLE S/N/N C-SET

In this chapter, theory for full counting statistics (FCS) in two coupled capasitively coupled single electron transistors (C-SETs) consisting of a common superconductor/nomal conducting island with gate electrode /normal conducting drain. Hereafter we call it *double S/N/N C-SET*. This is the basic ultra-small solid state entangler (SSE) and can be called the Cooper pair splitter (CPS) made of metallic tunnel junctions. Based on the theory, we discuss currents and cross correlation of current noise in superconducting subgap region.

3.1 Model and Hamiltonian

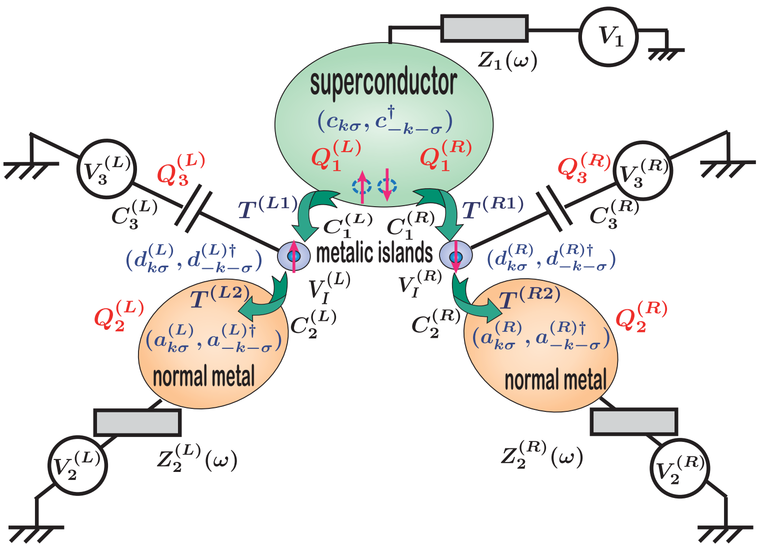


Fig. 3.1 Double S/N/N C-SET structures with external impedances.

Let us consider the double S/N/N C-SET as shown in Fig.3.1. This solid state entangler (SSE) is equivalent to the Cooper pair splitter experimentally reported [7, 9, 11, 12]. Hamiltonian of the system is given as

$$K = K_0 + H_\mathrm{T} \ , \tag{3.1}$$

where non-perturbative term K_0 is Hamiltonian of electrodes and is described as

$$K_0 = K_\mathrm{es} + H_\mathrm{em} \ , \tag{3.2}$$

and perturbative term H_T is tunnel Hamiltonian. Where the Hamiltonian of electrodes Kes describing the common superconductor(c), normal metal electrodes(a_α, ($\alpha = \mathrm{L,R}$)), and normal metal islands(d_α) are given by

$$\begin{aligned} K_\mathrm{es} = \frac{1}{2}\sum_{\boldsymbol{k}\sigma} \hat{\Psi}^{(c)\dagger}_{\boldsymbol{k}\sigma}\left[\xi^{(c)}_{\boldsymbol{k}}\hat{\tau}_3 - \Delta\hat{\tau}_+ - \Delta^*\hat{\tau}_-\right]\hat{\Psi}^{(c)}_{\boldsymbol{k}\sigma} \\ +\frac{1}{2}\sum_{\alpha=\mathrm{L,R}}\sum_{\boldsymbol{k}\sigma}\hat{\Psi}^{(d_\alpha)\dagger}_{\boldsymbol{k}\sigma}\xi^{(d_\alpha)}_{\boldsymbol{k}}\hat{\tau}_3\hat{\Psi}^{(d_\alpha)}_{\boldsymbol{k}\sigma} \\ +\frac{1}{2}\sum_{\alpha=\mathrm{L,R}}\sum_{\boldsymbol{k}\sigma}\hat{\Psi}^{(a_\alpha)\dagger}_{\boldsymbol{k}\sigma}\xi^{(\alpha)}_{\boldsymbol{k}}\hat{\tau}_3\hat{\Psi}^{(a_\alpha)}_{\boldsymbol{k}\sigma} \ , \end{aligned} \tag{3.3}$$

where $\sigma = (1, -1)$ or $(\uparrow, \downarrow)$, and Δ is the energy gap of superconducting electrodes. We employ the symbol $\hat{}$ for denoting matrices or spinors in Nambu-Gor′kov space. $\hat{\tau}_3$ and $\hat{\tau}_\pm = 1/\sqrt{2}(\hat{\tau}_1 \pm \mathrm{i}\hat{\tau}_2)$ is the Pauli matrices and Nambu-Gorkov spinors are defined as

$$\hat{\Psi}^{(c)}_{\boldsymbol{k}\sigma} = \begin{pmatrix} c_{\boldsymbol{k}\sigma} \\ c^\dagger_{-\boldsymbol{k}-\sigma} \end{pmatrix}, \hat{\Psi}^{(d_\alpha)}_{\boldsymbol{k}\sigma} = \begin{pmatrix} d^{(\alpha)}_{\boldsymbol{k}\sigma} \\ d^{(\alpha)\dagger}_{-\boldsymbol{k},-\sigma} \end{pmatrix}, \hat{\Psi}^{(a_\alpha)}_{\boldsymbol{k}\sigma} = \begin{pmatrix} a^{(\alpha)}_{\boldsymbol{k}\sigma} \\ a^{(\alpha)\dagger}_{-\boldsymbol{k},-\sigma} \end{pmatrix} . \tag{3.4}$$

H_em is the electromagnetic part of the Hamiltonian and describes electromagnetic environment effect caused by environmental impedances. In the C-SET theory [5, 23, 28, 41–45], charges $Q^{(\alpha)}_i$ (See Fig.3.1) induced on the electrodes are all treated as macroscopic quantum variables. Therefore, we introduce phase variables $\varphi^{(\alpha)}_i$, which satisfy the boson type commutation relations with charges, *i.e.*

$$[Q^{(\alpha)}_i, \varphi^{(\alpha')}_{i'}] = \mathrm{i}\hbar\delta_{\alpha,\alpha'}\delta_{i,i'} \ . \ \ (i, i' = 1, 2, 3) \tag{3.5}$$

In the absence of Ohmic resistance, H_em can be exactly treated and is shown to be decoupled into two part (See **Appendix A**):

$$H_\mathrm{em} = H_\mathrm{env} + H_\mathrm{c} \ , \tag{3.6}$$

$$H_\mathrm{env} = \sum_\alpha\sum_{j=1}^{2}\left\{\frac{(\omega^{(\alpha)}_j/\omega^{(\alpha)}_L)^2}{2L^{(\alpha)}_\Sigma}\varphi'^{(\alpha)2}_j + \frac{Q'^{(\alpha)2}_j}{2C^{(\alpha)}} - Q'^{(\alpha)}_j V'^{(\alpha)}_j\right\}, \tag{3.7}$$

$$H_\mathrm{c} = \sum_\alpha\left(\frac{q^{(\alpha)}}{e} - n^{(\alpha)}_\mathrm{c}\right)^2 U^{(\alpha)}, \tag{3.8}$$

where H_env and H_c describe environmental effect and charging effect in the island, respectively. $\omega^{(\alpha)}_j$ $(j = 1, 2)$ is the electromagnetic mode of α C-SET, and $q^{(\alpha)}$ is charges of α island. Diagonalized variables satisfy

$$[Q'^{(\alpha)}_i, \varphi'^{(\alpha')}_{i'}] = \mathrm{i}\hbar\delta_{\alpha,\alpha'}\delta_{i,i'} \ , \ \ (i = 1, 2) \tag{3.9}$$

$$[q^{(\alpha)}, \varphi'^{(\alpha')}_i] = \mathrm{i}\hbar\delta_{\alpha,\alpha'}\delta_{i,3} \ . \tag{3.10}$$

Relations between original phases and diagonalized ones are given as

$$\varphi_i^{(\alpha)} = \varphi_{i,\mathrm{env}}^{(\alpha)} + \varphi_{i,\mathrm{c}}^{(\alpha)} , \tag{3.11}$$

where

$$\varphi_{i,\mathrm{env}}^{(\alpha)} = \sum_{j=1,2} \sqrt{\kappa_i^{(\alpha)}} \eta_{ij}^{(\alpha)} \varphi_j^{\prime(\alpha)} , \tag{3.12}$$

$$\varphi_{i,\mathrm{c}}^{(\alpha)} = \sqrt{\kappa_i^{(\alpha)}} \eta_{i3}^{(\alpha)} \varphi_3^{\prime(\alpha)} , \tag{3.13}$$

are charging and environmental phases, respectively, and $\kappa_i^{(\alpha)} = C_i^{(\alpha)}/C_\Sigma^{(\alpha)}$ ($C_\Sigma^{(\alpha)} = \sum_{i=1}^3 C_i^{(\alpha)}$), and $\boldsymbol{\eta}^{(\alpha)}$ is the matrix which transforms a set of original charges, their canonical conjugate phases and electric potentials, ($Q_i^{(\alpha)}$, $\varphi_i^{(\alpha)}$, $V_i^{(\alpha)}$) into diagonalized one, ($Q_i^{\prime(\alpha)}$, $\varphi_i^{\prime(\alpha)}$, $V_i^{\prime(\alpha)}$). Furthermore,

$$U^{(\alpha)} = \frac{e^2}{2C_\Sigma^{(\alpha)}} , \tag{3.14}$$

$$n_\mathrm{c}^{(\alpha)} = -\sum_{j=1}^{3} \frac{C_j V_j}{e} + \frac{eV_\mathrm{I}^{(\alpha)}}{2U^{(\alpha)}} , \tag{3.15}$$

are, respectively, charging energy of the island and charge offset of C-SET on the α (=L, R) side. $n_\mathrm{c}^{(\alpha)}$, which contains chemical potential of α island, should be determined self-consistently by current continuity conditions. Note that perfect decoupling of H_em into H_env and H_c tells us that we have to treat two kinds of charges, *i.e.* continuously changeable charges $Q_i^{\prime(\alpha)}$ ($i = 1, 2$) with ability to fluctuate and quantized charge (island charge) $q^{(\alpha)}$, when we discuss effects caused by electromagnetic energy on the tunneling. In the C-SET theory electromagnetic environment and charging effects are, respectively, described in terms of correlation functions of phases $\varphi_{i'}^{\prime(\alpha)}$ (conjugate to $Q_i^{\prime(\alpha)}$) and $\varphi_3^{\prime(\alpha)}$ (conjugate to $q^{(\alpha)}$). In the presence of Ohmic resistance, although description of charging effect in terms of H_c remains unchanged, we cannot treat electromagnetic environment effect due to ohmic dissipation by Hamiltonian scheme like Eq.(3.7). We will discuss on this treatment in the next section.

Quite naturally tunneling Hamiltonian H_T is phase dependent and is expressed in terms of Nambu-Gor′kov spinors as

$$H_\mathrm{T} = \sum_\alpha \sum_{\boldsymbol{kk}'\sigma} \left[\hat{\Psi}_{\boldsymbol{k}\sigma}^{(c)\dagger} \hat{T}_{\boldsymbol{kk}'}^{(\alpha 1)} \hat{\Psi}_{\boldsymbol{k}'\sigma}^{(d_{\alpha 1})} + \hat{\Psi}_{\boldsymbol{k}\sigma}^{(a_\alpha)\dagger} \hat{T}_{\boldsymbol{kk}'}^{(\alpha 2)} \hat{\Psi}_{\boldsymbol{k}'\sigma}^{(d_{\alpha 2})} \right] , \tag{3.16}$$

where

$$\hat{\Psi}_{\boldsymbol{k}\sigma}^{(d_{\alpha i})} = \begin{pmatrix} d_{\boldsymbol{k}\sigma}^{(\alpha)} \cdot \mathrm{e}^{-\mathrm{i}\frac{e}{\hbar}\varphi_i^{(\alpha)}} \\ d_{-\boldsymbol{k},-\sigma}^{(\alpha)\dagger} \cdot \mathrm{e}^{\mathrm{i}\frac{e}{\hbar}\varphi_i^{(\alpha)}} \end{pmatrix} , \tag{3.17}$$

$$\hat{T}_{\boldsymbol{kk}'}^{(\alpha i)}(t) = \begin{pmatrix} T_{\boldsymbol{kk}'}^{(\alpha i)} \mathrm{e}^{\mathrm{i}\frac{e}{\hbar}[V_i^{(\alpha)} - V_\mathrm{I}^{(\alpha)}]t} & 0 \\ 0 & -T_{\boldsymbol{kk}'}^{(\alpha i)*} \mathrm{e}^{-\mathrm{i}\frac{e}{\hbar}[V_i^{(\alpha)} - V_\mathrm{I}^{(\alpha)}]t} \end{pmatrix} . \tag{3.18}$$

3.2 Description of Effect of Ohmic Resistance

In prior to the FCS formulation let us drop in at the discussion on how to describe electromagnetic environment effect due to Ohmic resistance.

For simplicity, let us consider the lowest order tunneling process. Since we defined phase dependent creation and annihilation operators of island electrons (see Eq.(3.17)), the correlation functions of phases appear Green's functions for island electrons. The Keldysh Green's function defined by Nambu-Gor'kov spinor for island electrons of α C-SET $\check{g}_{d_{\alpha i}}$ is defined as

$$\begin{aligned}\check{g}_{d_{\alpha i}}(\boldsymbol{k};t,t') &\equiv -\mathrm{i}\left\langle T_C \hat{\Psi}^{(d_{\alpha i})}_{\boldsymbol{k}\boldsymbol{\sigma}}(t)\hat{\Psi}^{(d_{\alpha i})\dagger}_{\boldsymbol{k}\boldsymbol{\sigma}}(t')\right\rangle_0 \\ &= \begin{pmatrix}\check{g}_{d_{\alpha i}11}(\boldsymbol{k};t,t') & 0 \\ 0 & \check{g}_{d_{\alpha i}22}(\boldsymbol{k};t,t')\end{pmatrix},\end{aligned} \tag{3.19}$$

where $\check{g}_{d_{\alpha i}11}(\boldsymbol{k};t,t')$ and $\check{g}_{d_{\alpha i}22}(\boldsymbol{k};t,t')$ are the diagonal components of Nambu-Gor'kov space, which are also 2×2 matrix in Schwinger-Keldysh space and are given as

$$\begin{aligned}\check{g}_{d_{\alpha i}11}(\boldsymbol{k};t,t') =& \left\langle T_C d_{\boldsymbol{k}\sigma}\alpha(t) d^\dagger_{\boldsymbol{k}\sigma}\alpha(t')\right\rangle_0 \left\langle T_C \mathrm{e}^{\mathrm{i}\frac{e}{\hbar}[\varphi^{(\alpha)}_{i,\mathrm{env}}(t)+\varphi^{(\alpha)}_{i,\mathrm{c}}(t)]}\mathrm{e}^{-\mathrm{i}\frac{e}{\hbar}[\varphi^{(\alpha)}_{i,\mathrm{env}}(t')+\varphi^{(\alpha)}_{i,\mathrm{c}}(t')]}\right\rangle_0 \\ =& \check{g}_{d_\alpha 11}(\boldsymbol{k};t,t')\ \check{\mathcal{F}}^{(\alpha i)}_{+-,\mathrm{env}}(t,t')\ \check{\mathcal{F}}^{(\alpha i)}_{+-,\mathrm{c}}(t,t'),\end{aligned} \tag{3.20}$$

$$\begin{aligned}\check{g}_{d_{\alpha i}22}(\boldsymbol{k};t,t') =& \left\langle T_C d^\dagger_{\boldsymbol{k}\sigma}\alpha(t) d_{\boldsymbol{k}\sigma}\alpha(t')\right\rangle_0 \left\langle T_C \mathrm{e}^{-\mathrm{i}\frac{e}{\hbar}[\varphi^{(\alpha)}_{i,\mathrm{env}}(t)+\varphi^{(\alpha)}_{i,\mathrm{c}}(t)]}\mathrm{e}^{\mathrm{i}\frac{e}{\hbar}[\varphi^{(\alpha)}_{i,\mathrm{env}}(t')+\varphi^{(\alpha)}_{i,\mathrm{c}}(t')]}\right\rangle_0 \\ =& \check{g}_{d_\alpha 22}(\boldsymbol{k};t,t')\ \check{\mathcal{F}}^{(\alpha i)}_{-+,\mathrm{env}}(t,t')\ \check{\mathcal{F}}^{(\alpha i)}_{-+,\mathrm{c}}(t,t'),\end{aligned} \tag{3.21}$$

where $\check{\mathcal{F}}^{(\alpha i)}_{\lambda\lambda',\mathrm{env}}(t,t')$ and $\check{\mathcal{F}}^{(\alpha i)}_{\lambda\lambda',\mathrm{c}}(t,t')$ are correlation functions of phases which describe the electromagnetic environment effect and the charging effect, respectively, and are defined by

$$\check{\mathcal{F}}^{(\alpha i)}_{\lambda\lambda',\mathrm{env}}(t-t') \equiv -\mathrm{i}\langle\, T_C \mathrm{e}^{\mathrm{i}\lambda\frac{e}{\hbar}\varphi^{(\alpha)}_{i,\mathrm{env}}(t)}\mathrm{e}^{\mathrm{i}\lambda'\frac{e}{\hbar}\varphi^{(\alpha)}_{i,\mathrm{env}}(t')}\rangle_0, \tag{3.22}$$

$$\check{\mathcal{F}}^{(\alpha i)}_{\lambda\lambda',\mathrm{c}}(t-t') \equiv -\mathrm{i}\langle T_C \mathrm{e}^{\mathrm{i}\lambda\frac{e}{\hbar}\varphi_{i,\mathrm{c}^{(\alpha)}(t)}}\mathrm{e}^{\mathrm{i}\lambda'\frac{e}{\hbar}\varphi^{(\alpha)}_{i,\mathrm{c}}(t')}\rangle_0 = \mathcal{F}^{(\alpha i)}_{\lambda-\lambda,\mathrm{c}}(t-t')\delta_{\lambda',-\lambda}\,. \tag{3.23}$$

$$(\lambda,\lambda' = +\mathrm{or}-) \tag{3.24}$$

no ohmic resistance

Let us first calculate $\check{\mathcal{F}}^{(\alpha i)}_{\lambda\lambda',\mathrm{env}}(t-t')$ *in the absence of ohmic resistance.* Introducing

$$b^{(\alpha)}_j = \sqrt{\frac{\omega^{(\alpha)}_j C^{(\alpha)}}{2\hbar}}\varphi'^{(\alpha)} + \mathrm{i}\frac{1}{\sqrt{2\hbar\omega^{(\alpha)}_j C^{(\alpha)}}}\big(Q'^{(\alpha)} - C^{(\alpha)}V'^{(\alpha)}_j\big), \tag{3.25}$$

which satisfies $[b^{(\alpha)}_j, b^{(\alpha')\dagger}_{j'}] = \delta_{jj'}\delta_{\alpha\alpha'}$, we first note that Eq.(3.7) is rewritten as

$$H_{\mathrm{env}} = \sum_\alpha\sum_{j=1}^{2}\hbar\omega^{(\alpha)}_j\left(b^{(\alpha)\dagger}_j b^{(\alpha)}_j + \frac{1}{2}\right)\,. \tag{3.26}$$

Making use of Campbell-Baker-Hausdorff formula

$$\mathrm{e}^{A+B} = \mathrm{e}^A\mathrm{e}^B\mathrm{e}^{-[A,B]/2} = \mathrm{e}^B\mathrm{e}^A\mathrm{e}^{[A,B]/2}\,, \tag{3.27}$$

with $[A,[A,B]]=[B,[A,B]]$, we obtain

$$\begin{pmatrix} \mathcal{F}^{(\alpha i)-+}_{+-,\mathrm{env}}(t-t') \\ \mathcal{F}^{(\alpha i)+-}_{+-,\mathrm{env}}(t-t') \end{pmatrix} = \begin{pmatrix} \exp\left(J^{(\alpha i)-+}_{+-}(t-t')\right) \\ \exp\left(J^{(\alpha i)+-}_{+-}(t-t')\right) , \end{pmatrix} \tag{3.28}$$

where

$$\begin{pmatrix} J^{(\alpha i)-+}_{+-}(t-t') \\ J^{(\alpha i)+-}_{+-}(t-t') \end{pmatrix} = \sum_j \frac{\kappa_i^{(\alpha)}\eta_{ij}^{(\alpha)2}E_c^{(\alpha)}}{\hbar\omega_j^{(\alpha)}} \times \left[\coth\frac{\beta\hbar\omega_j^{(\alpha)}}{2}\left(\cos\omega_j^{(\alpha)}(t-t')-1\right)\pm \mathrm{i}\sin\omega_j^{(\alpha)}(t-t')\right] \tag{3.29}$$

and charging energy of α C-SET is given by $E_c^{(\alpha)}=e^2/2C^{(\alpha)}$.

effect of ohmic resistance

As is explained in Sec.2.2.2, one of the main effects of the electromagnetic environmental impedance is to cause fluctuation in *continuous* charge. Charge fluctuation in tunnel junction with environment inductance $Z_L(\omega)=\mathrm{i}\omega L$ can be also obtained not only quantum mechanical treatment but phenomenological circuit theory. The charge fluctuation discussed in Eq.(2.13), for example, can be obtained as

$$\begin{aligned}\langle\delta Q^2\rangle &= \frac{e^2}{2}\frac{\hbar\omega_L}{E_c}\left\{\frac{1}{\exp(\beta\hbar\omega_L)-1}+\frac{1}{2}\right\} \\ &= 2\left(\frac{\hbar C}{e}\right)^2\int_0^\infty \frac{\mathrm{d}\omega}{\omega}\frac{\mathrm{Re}[Z_t(\omega)]}{R_q}\coth\left(\frac{1}{2}\beta\hbar\omega\right) ,\end{aligned} \tag{3.30}$$

with the following total impedance

$$Z_t(\omega)=\frac{1}{\mathrm{i}\omega C+Z_L(\omega)^{-1}}=\frac{1}{C}\frac{\mathrm{i}\omega}{\omega_L^2-(\omega-\mathrm{i}\delta)^2} . \quad (\delta\to+0) \tag{3.31}$$

In this simplest case the two-body correlation of environmental phase takes a form

$$\mathcal{F}^{+-}_{+-,\mathrm{env}}(t-t')=\mathrm{e}^{J^{+-}(t-t')} , \tag{3.32}$$

where

$$\begin{aligned}J^{+-}_{+-}(t-t') &= \kappa^2\frac{E_\mathrm{c}}{\hbar\omega_L}\left[\coth\left(\frac{1}{2}\beta\hbar\omega_L\right)(\cos\omega_L(t-t')-1)-\mathrm{i}\sin\omega_L(t-t')\right] \\ &= \int_0^\infty \frac{\mathrm{d}\omega}{\omega}\frac{\mathrm{Re}[Z_t(\omega)]}{R_q} \\ &\quad\times\left\{\coth\left(\frac{\beta\hbar\omega}{2}\right)[\cos(\omega(t-t'))-1]\pm\mathrm{i}\sin(\omega(t-t')\right\} .\end{aligned} \tag{3.33}$$

Exactly the same physics appears in both quantities. The effect of ohmic resistance cannot be treated in the Hamiltonian scheme. Furthermore, even Caldeira-Leggett theory [46] only open the way to treat *energy dissipation in quantum mechanics* as far as physical systems stay in one-particle picture. Nevertheless result stated above enables us to treat the problem phenomenologically [47] in the spirit of Cardeira-Leggett. Therefore, in the case of double S/N/N C-SET, we evaluate the environmental phase correlation as

$$\begin{pmatrix} J^{(\alpha i)-+}(t-t') \\ J^{(\alpha i)+-}(t-t') \end{pmatrix} = \int_0^\infty \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{Re}[Z_t^{(\alpha)}(\omega)]}{R_q} \times \left\{ \coth\left(\frac{\beta\hbar\omega}{2}\right) [\cos(\omega(t-t'))-1] \pm \mathrm{i}\sin(\omega(t-t')) \right\}, \tag{3.34}$$

with total impedance as shown in Fig.3.2

$$Z_t^{(\alpha)}(\omega) = \frac{1}{\mathrm{i}\omega C_\Sigma^{(\alpha)} + Z^{(\alpha)-1}(\omega)}, \tag{3.35}$$

where

$$Z^{(\alpha)}(\omega) = \frac{Z_1(\omega)}{2} + Z_2^{(\alpha)}(\omega), \tag{3.36}$$

$$Z_1(\omega) = \mathrm{i}\omega L_1 + R_1, \tag{3.37}$$

$$Z_2^{(\alpha)}(\omega) = \mathrm{i}\omega L_2^{(\alpha)} + R_2^{(\alpha)}. \tag{3.38}$$

In order to understand physics due to ohmic resistance, it is instructive to express $Z_t^{(\alpha)}(\omega)$ in terms of *quality factor* $\mathcal{Q}^{(\alpha)}$ for α C-SET which characterize the energy

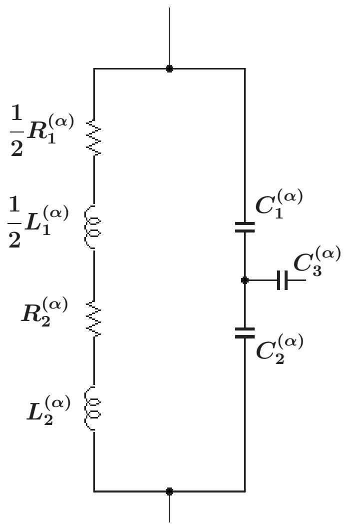


Fig. 3.2 Impedance of α C-SET of double S/N/N C-SET. Environmental inductance $\mathrm{i}\omega L$ and ohmic resistance R are introduced in series and are coupled to C-SET in parallel. Environmental impedance of superconductor is taken into account equally in both C-SETs.

dissipation in resonant oscillation and is defined by

$$\mathcal{Q}^{(\alpha)} \equiv \frac{\omega_R^{(\alpha)}}{\omega_L^{(\alpha)}} , \tag{3.39}$$

$$\omega_R^{(\alpha)} \equiv \frac{1}{\left(\frac{R_1}{2} + R_2^{(\alpha)}\right) \cdot C_\Sigma^{(\alpha)}} . \tag{3.40}$$

Total impedance is then expressed as

$$Z_t^{(\alpha)}(\omega) = \left(\frac{R_1}{2} + R_2^{(\alpha)}\right) \cdot \frac{1 + \mathrm{i}\left[\mathcal{Q}^{(\alpha)}\right]^2 \dfrac{\omega}{\omega_R^{(\alpha)}}}{1 + \mathrm{i}\dfrac{\omega}{\omega_R^{(\alpha)}} - \left[\mathcal{Q}^{(\alpha)}\right]^2 \left[\dfrac{\omega}{\omega_R^{(\alpha)}}\right]^2} . \tag{3.41}$$

$\mathcal{Q}^{(\alpha)} \to /(\infty)$ corresponds to large energy dissipation (no ohmic resistance) case.

3.3 Full Counting Statistics

3.3.1 Counting field and Hamiltonian for FCS

Following prescription stated in Sec.2.6.2, let us introduce counting field. In the case double S/N/N C-SET, we have to pay special attention to the superconducting electrode, since superconducting electrode is a common electrode for double C-SET and the BCS Hamiltonian never conserves particle number. In order to introduce counting fields in a relevant way, we introduce the following generalized unitary transformation:

$$K(\{\chi_\nu(\tau);\tau\}) \equiv M(\{\chi_\nu(\tau)\};\tau)^\dagger \cdot K(\tau) \cdot M(\{\chi_\nu(\tau);\tau\}) \tag{3.42}$$

where

$$M(\{\chi_\nu(\tau)\};\tau) \equiv \exp\left\{\frac{\mathrm{i}}{2}\left[\chi_c(\tau)N_c(\tau) + \sum_{\alpha=\mathrm{L,R}}\sum_{\nu=d_\alpha,a_\alpha} \chi_\nu(\tau)N_\nu(\tau)\right]\right\} . \tag{3.43}$$

Here we assigned notations c, d_α, and a_α to counting fields and the numbers of electrons in superconducting, islands, and normal drain electrodes on α C-SET. Resulting counting field dependent Hamiltonian takes then the form, for example on C_- ;

$$\begin{aligned} K_0(\chi_c) = \frac{1}{2}\sum_{\boldsymbol{k}\sigma} \hat{\Psi}_{\boldsymbol{k}\sigma}^{(c)\dagger}\left[\xi_{\boldsymbol{k}}^{(c)}\hat{\tau}_3 - \mathrm{e}^{\mathrm{i}\chi_c\hat{\tau}_3}\Delta\hat{\tau}_+ - \mathrm{e}^{-\mathrm{i}\chi_c\hat{\tau}_3}\Delta^*\hat{\tau}_-\right]\hat{\Psi}_{\boldsymbol{k}\sigma}^{(c)} \\ + \frac{1}{2}\sum_{\alpha=\mathrm{L,R}}\sum_{\boldsymbol{k}\sigma} \hat{\Psi}_{\boldsymbol{k}\sigma}^{(d_\alpha)\dagger}\xi_{\boldsymbol{k}}^{(d_\alpha)}\hat{\tau}_3\hat{\Psi}_{\boldsymbol{k}\sigma}^{(d_\alpha)} \\ + \frac{1}{2}\sum_{\alpha=\mathrm{L,R}}\sum_{\boldsymbol{k}\sigma} \hat{\Psi}_{\boldsymbol{k}\sigma}^{(a_\alpha)\dagger}\xi_{\boldsymbol{k}}^{(\alpha)}\hat{\tau}_3\hat{\Psi}_{\boldsymbol{k}\sigma}^{(a_\alpha)} , \end{aligned} \tag{3.44}$$

and

$$\begin{aligned} H_{\mathrm{T}}(\{\chi\}) = \sum_{\alpha}\sum_{\boldsymbol{k}\boldsymbol{k}'\sigma} \Big\{ \hat{\Psi}_{\boldsymbol{k}\sigma}^{(c)\dagger}\hat{T}_{\boldsymbol{k}\boldsymbol{k}'}^{(\alpha 1)}\mathrm{e}^{\frac{\mathrm{i}}{2}(\chi_c - \chi_{d_\alpha})\hat{\tau}_3}\hat{\Psi}_{\boldsymbol{k}'\sigma}^{(d_{\alpha 1})} \\ + \hat{\Psi}_{\boldsymbol{k}\sigma}^{(a_\alpha)\dagger}\hat{T}_{\boldsymbol{k}\boldsymbol{k}'}^{(\alpha 2)}\mathrm{e}^{\frac{\mathrm{i}}{2}(\chi_{a_\alpha} - \chi_{d_\alpha})\hat{\tau}_3}\hat{\Psi}_{\boldsymbol{k}'\sigma}^{(d_{\alpha 2})} \Big\} . \end{aligned} \tag{3.45}$$

Counting field χ_c appearing in off-diagonal terms of BCS Hamiltonian is irrelevant to FCS, since these terms are not concerned with tunneling event. Furthermore, this has no effect on statistical average which concerns superconducting state. Combinations of counting fields $\chi_c - \chi_{d_\alpha}$ and $\chi_{a_\alpha} - \chi_{d_\alpha}$, on the other hand, are measures of changes in electron numbers which caused by tunnelings from islands d_α to electrodes c and a_α, respectively. Therefore, according to the discussion in Sec.2.6.2 it, it is plausible to identify

$$\chi_\alpha \equiv \chi_c - \chi_{d_\alpha} = -(\chi_{a_\alpha} - \chi_{d_\alpha}) \ , \tag{3.46}$$

with the effective counting fields for the α C-SET. Finally the starting point of FCS for the double S/N/N C-SET is given by the following counting field dependent tunneling Hamiltonian

$$\begin{aligned} H_{\mathrm{T},C}(\{\chi_\alpha(\tau)\};\tau) = \sum_{\alpha=\mathrm{L,R}} \sum_{\boldsymbol{k}\boldsymbol{k}'\sigma} \Big\{ & \hat{\Psi}^{(c)\dagger}_{\boldsymbol{k}\sigma}(\tau) \hat{T}^{(\alpha 1)}_{\boldsymbol{k}\boldsymbol{k}'}(\tau) \mathrm{e}^{\frac{\mathrm{i}}{2}\chi_\alpha(\tau)\hat{\tau}_3} \hat{\Psi}^{(d_{\alpha 1})}_{\boldsymbol{k}'\sigma}(\tau) \\ & + \hat{\Psi}^{(a_\alpha)\dagger}_{\boldsymbol{k}\sigma}(\tau) \hat{T}^{(\alpha 2)}_{\boldsymbol{k}\boldsymbol{k}'}(\tau) \mathrm{e}^{-\frac{\mathrm{i}}{2}\chi_\alpha(\tau)\hat{\tau}_3} \hat{\Psi}^{(d_{\alpha 2})}_{\boldsymbol{k}'\sigma}(\tau) \Big\} \ , \end{aligned} \tag{3.47}$$

where

$$\chi_\alpha(\tau) \equiv \begin{cases} \chi_\alpha(t_-) = \ \ \chi_\alpha & \text{for } \tau = t_- \ (\tau \text{ on } C_-) \\ \chi_\alpha(t_+) = -\chi_\alpha & \text{for } \tau = t_+ \ (\tau \text{ on } C_+) \end{cases} \ , \tag{3.48}$$

for

$$\theta\left(t_\mp \pm \frac{T_0}{2}\right) \theta\left(-t_\mp \pm \frac{T_0}{2}\right) \ . \tag{3.49}$$

As is discussed in Sec.2.6.2, the counting field dependent time evolution operator and characteristic function (moment generation function) are then defined in terms of $H_{\mathrm{T},C}(\{\chi_\alpha(\tau)\})$:

$$\begin{aligned} U_C(\{\chi_\alpha\};T_0) &\equiv T_C \exp\left\{ -\frac{\mathrm{i}}{\hbar} \int_C \mathrm{d}t \ H_{\mathrm{T},C}(\chi_\alpha(\tau);\tau) \right\} \\ &\equiv U_+\left(\{\chi_\alpha\}; -\frac{T_0}{2}, \frac{T_0}{2} \right) \cdot U_-\left(\{\chi_\alpha\}; \frac{T_0}{2}, -\frac{T_0}{2} \right) \ , \end{aligned} \tag{3.50}$$

$$Z(\{\chi_\alpha\};T_0) \equiv \Big\langle U_C(\{\chi_\alpha\};T_0) \Big\rangle_{0,T_0} \ . \tag{3.51}$$

3.3.2 Cumulant generating function

The cumulant generating function (CGF) $\Omega(\{\chi_\alpha\})$ for double S/N/N C-SET is defined in long time average by

$$\Omega(\{\chi_\alpha\}) \equiv \lim_{T_0\to\infty} \frac{1}{T_0} \log \left\langle T_C \exp\Big\{ -\frac{\mathrm{i}}{\hbar} \int_C \mathrm{d}\tau \ H_{\mathrm{T},C}(\{\chi_\alpha(\tau)\},\tau) \Big\} \right\rangle_{0,T_0} \ . \tag{3.52}$$

We ask the readers to refer to **APPENDIX B** for the derivation of CGF, we only show here the explicit expression for CGF:

$$\begin{aligned} \Omega(\{\chi_\alpha\}) = \frac{1}{2} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \, \mathrm{Tr}_{SK\otimes NG} \Bigg\{ & \log \left[\frac{\check{1} - \check{g}_c(\omega) \check{\Sigma}_c(\{\chi_\alpha\},\omega)/\hbar}{\check{1} - \check{g}_c(\omega) \check{\Sigma}_c(0;\omega)/\hbar} \right] \\ & + \sum_{\alpha=\mathrm{L,R}} \log \left[\frac{\check{1} - \check{g}_{a_\alpha}(\omega) \check{\Sigma}_{a_\alpha}(\chi_\alpha,\omega)/\hbar}{\check{1} - \check{g}_{a_\alpha}(\omega) \check{\Sigma}_{a_\alpha}(0;\omega)/\hbar} \right] \Bigg\} \ , \end{aligned} \tag{3.53}$$

where $\mathrm{Tr}_{SK\otimes NG}$ denotes traces over both Schwinger-Keldysh and Nambu-Gor'kov spaces. Here

$$\begin{aligned}\check{\Sigma}_c(\{\chi_\alpha\},\omega) =&\sum_\alpha \check{T}_{cd_\alpha}(\chi_\alpha)\check{g}_{d_{\alpha 1}}(\omega)\check{T}_{d_\alpha c}(\chi_\alpha)/\hbar\ ,\\ \check{\Sigma}_{a_\alpha}(\chi_\alpha,\omega) =&\check{T}_{a_\alpha d_\alpha}(\chi_\alpha)\check{g}_{d_{\alpha 2}}(\omega)\check{T}_{d_\alpha a_\alpha}(\chi_\alpha)/\hbar\ ,\end{aligned} \tag{3.54}$$

are self-energies of corresponding Keldysh Green's functions defined by Nambu spinors as

$$\begin{aligned}\check{G}_{AB}(\{\chi_\alpha\},\omega) \equiv& \sum_{\boldsymbol{k},\boldsymbol{k}'}\int_{-\infty}^{\infty} \mathrm{d}t\ \mathrm{e}^{\mathrm{i}\omega(t-t')}\Big(-\mathrm{i}\langle\ T_C\hat{\Psi}^{(A)}_{\boldsymbol{k}\sigma}(t)\hat{\Psi}^{(B)\dagger}_{\boldsymbol{k}'\sigma}(t')\ \rangle_{K(\{\chi_\alpha\})}\Big)\\ =& \ \check{g}_A(\omega)\delta_{A,B} + \check{g}_A(\omega)\check{\Sigma}_A(\{\chi_\alpha\},\omega)\check{G}_{AB}(\{\chi_\alpha\},\omega),\end{aligned} \tag{3.55}$$

for electrons in superconductor ($A, B = c$) and normal metallic islands ($A, B = d_\alpha$) normal conducting drains ($A, B = a_\alpha$). Concerning tunnel matrix element, we omitted the wave vector dependence, and redefined as counting field dependent tunnel matrix element in Keldysh space $\check{T}_{AB}$ as

$$\check{T}_{cd_\alpha}(\chi_\alpha) \equiv \begin{pmatrix} \hat{T}_{cd_\alpha}\mathrm{e}^{\frac{\mathrm{i}}{2}\chi_\alpha\hat{\tau}_3}, & 0\\ 0, & -\hat{T}_{cd_\alpha}\mathrm{e}^{-\frac{\mathrm{i}}{2}\chi_\alpha\hat{\tau}_3}\end{pmatrix}, \tag{3.56}$$

$$\check{T}_{a_\alpha d_\alpha}(\chi_\alpha) \equiv \begin{pmatrix} \hat{T}_{a_\alpha d_\alpha}\mathrm{e}^{-\frac{\mathrm{i}}{2}\chi_\alpha\hat{\tau}_3}, & 0\\ 0, & -\hat{T}_{a_\alpha d_\alpha}\mathrm{e}^{\frac{\mathrm{i}}{2}\chi_\alpha\hat{\tau}_3}\end{pmatrix}, \tag{3.57}$$

with $\check{T}_{AB}(\chi_\alpha) = \check{T}^*_{BA}(\chi_\alpha)$, where $\hat{T}_{\nu\nu'} = \hat{T}^*_{\nu'\nu}$ is the 2×2 tunneling matrix of junction between ν and ν' electrodes which describes superconductor ($\nu = c$), normal metallic islands ($\nu = d_\alpha$) and normal conducting drains ($\nu = a_\alpha$). Note that $\check{G}_{AB}(\{\chi_\alpha\},\omega)$ is 2×2 matrix in Keldysh space:

$$\check{G}_{AB}(\{\chi_\alpha\},\omega) = \begin{pmatrix} \hat{G}^{--}_{AB}(\{\chi_\alpha\},\omega) & \hat{G}^{-+}_{AB}(\{\chi_\alpha\},\omega)\\ \hat{G}^{+-}_{AB}(\{\chi_\alpha\},\omega) & \hat{G}^{++}_{AB}(\{\chi_\alpha\},\omega)\end{pmatrix}, \tag{3.58}$$

each component of which is 2×2 matrix in Nambu space, say,

$$\hat{G}^{\lambda\lambda'}_{AB}(\{\chi_\alpha\},\omega) = \begin{pmatrix} G^{\lambda\lambda'}_{AB,11}(\{\chi_\alpha\},\omega) & G^{\lambda\lambda'}_{AB,12}(\{\chi_\alpha\},\omega)\\ G^{\lambda\lambda'}_{AB,21}(\{\chi_\alpha\},\omega) & G^{\lambda\lambda'}_{AB,22}(\{\chi_\alpha\},\omega),\end{pmatrix}, \tag{3.59}$$

$(\lambda, \lambda' = -$ or $+)$. In what follows, we denote $G_{AB}(0,\omega) = G_{AB}(\omega)$.

3.4 Currents in Double S/N/N C-SET

The current is obtained by first derivative of CGF:

$$\begin{aligned}I_\alpha =& (-e)\ \frac{\partial\Omega(\{\chi_\alpha\})}{\partial(\mathrm{i}\chi_\alpha)}\bigg|_{\{\chi_\alpha\}=0}\\ =& (-e)\frac{1}{2}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\mathrm{Tr}_{SK\otimes NG}\big[-\check{G}_{cc}(\{\chi_\alpha\},\omega)(\partial_{\mathrm{i}\chi_\alpha}\check{\Sigma}_c(\{\chi_\alpha\},\omega))\\ &-\check{G}_{a_\alpha a_\alpha}(\{\chi_\alpha\},\omega)(\partial_{\mathrm{i}\chi_\alpha}\check{\Sigma}_{a_\alpha}(\chi_\alpha,\omega))\big]|_{\{\chi_\alpha\}=0}\ ,\end{aligned} \tag{3.60}$$

where $\partial_{\mathrm{i}\chi_\alpha} \equiv \partial/\partial(\mathrm{i}\chi_\alpha)$. Noting that

$$\frac{\partial\check{\Sigma}_c(\{\chi_\alpha\},\omega)}{\partial(\mathrm{i}\chi_\alpha)} = \check{T}_{cd_\alpha}(\chi_\alpha)\begin{pmatrix} 0 & \hat{g}^{-+}_{d_{\alpha1}}(\omega) \\ -\hat{g}^{+-}_{d_{\alpha1}}(\omega) & 0 \end{pmatrix}\hat{\tau}_3\check{T}_{d_\alpha c}(\chi_\alpha), \tag{3.61}$$

$$\frac{\partial\check{\Sigma}_{a_\alpha}(\{\chi_\alpha\},\omega)}{\partial(\mathrm{i}\chi_\alpha)} = \check{T}_{a_\alpha d_\alpha}(\chi_\alpha)\begin{pmatrix} 0 & -\hat{g}^{-+}_{d_{\alpha2}}(\omega) \\ \hat{g}^{+-}_{d_{\alpha2}}(\omega) & 0 \end{pmatrix}\hat{\tau}_3\check{T}_{d_\alpha a_\alpha}(\chi_\alpha)\,. \tag{3.62}$$

Eq.(3.60) becomes

$$\begin{aligned} I_\alpha =&(-e)\frac{1}{2}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\mathrm{Tr}_{NG}\Big[-\hat{G}^{-+}_{cc}(\{\chi_\alpha\},\omega)\left(\frac{\partial\check{\Sigma}_c(\{\chi_\alpha\},\omega)}{\partial(\mathrm{i}\chi_\alpha)}\right)^{+-} \\ &-\hat{G}^{+-}_{cc}(\{\chi_\alpha\},\omega)\left(\frac{\partial\check{\Sigma}_c(\{\chi_\alpha\},\omega)}{\partial(\mathrm{i}\chi_\alpha)}\right)^{-+} - \hat{G}^{-+}_{a_\alpha a_\alpha}(\{\chi_\alpha\},\omega)\left(\frac{\partial\check{\Sigma}_{a_\alpha}(\chi_\alpha,\omega)}{\partial(\mathrm{i}\chi_\alpha)}\right)^{+-} \\ &-\hat{G}^{+-}_{a_\alpha a_\alpha}(\{\chi_\alpha\},\omega)\left(\frac{\partial\check{\Sigma}_{a_\alpha}(\chi_\alpha,\omega)}{\partial(\mathrm{i}\chi_\alpha)}\right)^{-+}\Big]\Big|_{\{\chi_\alpha\}=0}\,. \end{aligned} \tag{3.63}$$

Furthermore, utilizing the following exact relation

$$\begin{aligned} G^{\pm\mp}_{\nu\nu}(\omega) =& \left[1+\sum_\alpha G^{\mathrm{R}}_{\nu d_\alpha}(\omega)\frac{1}{\hbar}T_{d_\alpha\nu}\right]\hat{g}^{\pm\mp}_{\nu}(\omega)\left[\hat{\tau}_0+\sum_\alpha\frac{1}{\hbar}T_{\nu d_\alpha}G^{\mathrm{A}}_{d_\alpha\nu}(\omega)\right] \\ &+G^{\mathrm{R}}_{\nu\nu}(\omega)\left(\sum_\alpha\frac{1}{\hbar}T_{\nu d_\alpha}g^{\pm\mp}_{d_\alpha i}(\omega)\frac{1}{\hbar}T_{d_\alpha\nu}\right)G^{\mathrm{A}}_{\nu\nu}(\omega)\,, \\ &(\nu,i)\equiv(c,1)\ \text{ or }\ (a_\alpha,2) \end{aligned} \tag{3.64}$$

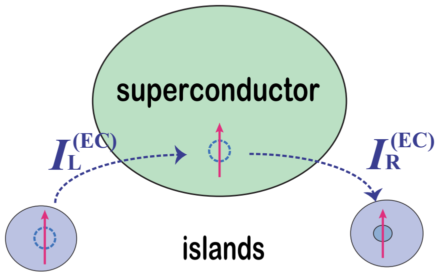
and taking trace in Nambu space, we arrive at the final expression for the currents:

$$\begin{aligned} I_\alpha =&\frac{e}{2}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\Big[\Big|\frac{1}{\hbar}T_{cd_\alpha}\Big|^2 \\ &\times\Big\{\big|1+\sum_{\nu=\mathrm{L,R}}\frac{1}{\hbar}T_{d_\nu c}G^{\mathrm{R}}_{cd_\nu 11}(\omega)\big|^2\left(g^{+-}_{c11}(\omega)g^{-+}_{d_{\alpha1}11}(\omega)-g^{-+}_{c11}(\omega)g^{+-}_{d_{\alpha1}11}(\omega)\right) \\ &-2\mathrm{Re}\Big[\big(1+\frac{1}{\hbar}\sum_{\nu=\mathrm{L,R}}T_{d_\nu c}G^{\mathrm{R}}_{cd_\nu 11}(\omega)\big)\frac{1}{\hbar}\sum_{\nu'=\mathrm{L,R}}T_{d_{\nu'}c}G^{\mathrm{A}}_{cd_{\nu'}21}\Big] \\ &\qquad\times\left(g^{+-}_{c12}(\omega)g^{-+}_{d_{\alpha1}11}(\omega)-g^{-+}_{c12}(\omega)g^{+-}_{d_{\alpha1}11}(\omega)\right) \\ &+\big|\sum_{\nu=\mathrm{L,R}}\frac{1}{\hbar}T_{cd_\nu}G^{\mathrm{R}}_{cd_{\nu1}12}(\omega)\big|^2\left(g^{+-}_{c12}(\omega)g^{-+}_{d_{\alpha1}11}(\omega)-g^{-+}_{c12}(\omega)g^{+-}_{d_{\alpha1}11}(\omega)\right) \\ &+\Big|\frac{1}{\hbar}T_{cd_{\bar{\alpha}}}\Big|^2\big|G^{\mathrm{R}}_{cc,11}(\omega)\big|^2\Big(g^{+-}_{d_{\alpha1},11}(\omega)g^{-+}_{d_{\bar{\alpha}1},11}(\omega)-g^{-+}_{d_{\alpha1},11}(\omega)g^{+-}_{d_{\bar{\alpha}1},11}(\omega)\Big) \\ &+\sum_{\alpha'=\mathrm{L,R}}\Big|\frac{1}{\hbar}T_{cd_{\alpha'}}\Big|^2\big|G^{\mathrm{R}}_{cc,12}(\omega)\big|^2\Big(g^{+-}_{d_{\alpha1},11}(\omega)g^{-+}_{d_{\alpha'1},22}(\omega)-g^{-+}_{d_{\alpha1},11}(\omega)g^{+-}_{d_{\alpha'1},22}(\omega)\Big)\Big\} \\ &-\Big|\frac{1}{\hbar}T_{a_\alpha d_\alpha}\Big|^2\Big|\ 1+T_{d_\alpha a_\alpha}G^{\mathrm{R}}_{c\tilde{d}_\alpha,11}(\omega)\ \Big|^2 \\ &\qquad\times\Big(g^{+-}_{d_{\alpha2},11}(\omega)g^{-+}_{a_\alpha,11}(\omega)-g^{-+}_{d_{\alpha2},11}(\omega)g^{+-}_{a_\alpha,11}(\omega)\Big)\Big]\,. \end{aligned} \tag{3.65}$$

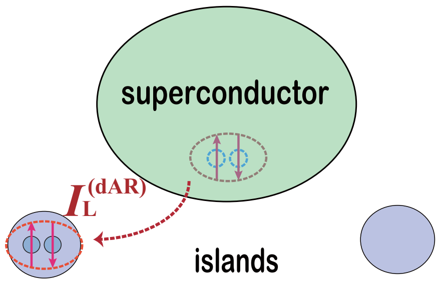
This formula describes all kinds of currents in double S/N/N C-SET due to quasi-particle, branch crossing, elastic co-tunneling(EC), crossed Andreev reflection (cAR), and direct Andreev reflection(dAR), if the current continuity condition is satisfied. In this case, the condition can be stated that the last term of Eq.(4.3) coincides with the rest terms of Eq.(4.3), which is nothing but the determination of optimum electric potential of the island of α C-SET. We are interested in the current which conveys quantum entanglement, we restrict ourselves to $I-V$ characteristics in the superconducting subgap region, *i.e.* in terms of electric potential of α island $V_{\rm I}^{(\alpha)}$ self-consistently determined by current continuity condition

$$e(V_{\rm I}^{(\alpha)} - V_1) < \Delta \, , \tag{3.66}$$

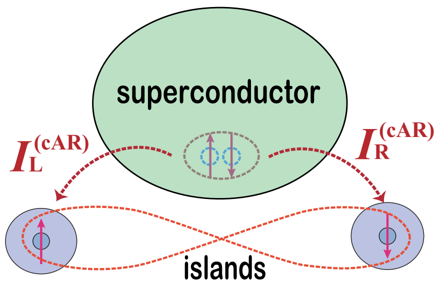
where currents due to EC, cAR and dAR survive.



(a) Elastic co-tunneling



(b) Direct Andreev reflection



(c) Crossed Andreev reflection

Fig. 3.3 Schematic diagrams for the tunneling mechanisms expected in superconducting subgap region.

3.5 Cross Correlation of Current Noise in Double S/N/N C-SET

Cross correlation of current noise is defined as

$$S_{\rm LR}(\omega) \equiv \int_{-\infty}^{\infty} {\rm d}(t-t')\,{\rm e}^{{\rm i}\omega(t-t')}\langle[\delta\mathcal{I}_{\rm L}(t),\delta\mathcal{I}_{\rm R}(t')]_+\rangle, \tag{3.67}$$

where $\mathcal{I}_\alpha(t)$ and

$$\delta\mathcal{I}_\alpha(t) = \mathcal{I}_\alpha(t) - \langle\mathcal{I}_\alpha(t)\rangle\ , \tag{3.68}$$

are, respectively, the current and current noise operators in α C-SET, and $[A,B]_+ \equiv AB+BA$ is the anti-commutation relation. Total cross correlation of current noise is obtained as the second derivative of CGF:

$$\begin{aligned} S_{\rm LR}(\omega=0) =&(-e)^2\frac{\partial^2\Omega(\{\chi_{\rm R}\})}{\partial({\rm i}\chi_\alpha)\partial({\rm i}\chi_{\rm L})}|_{\{\chi_\alpha\}=0} \\ =&-\frac{e^2}{2\hbar}\int_{-\infty}^{\infty}\frac{{\rm d}\omega}{2\pi} \\ &\times {\rm Tr}_{SK\otimes NG}\left\{\check{G}_{cc}(\{\chi_\alpha\},\omega)\frac{\partial\check{\Sigma}_c(\{\chi_\alpha\},\omega)}{\partial({\rm i}\chi_{\rm L})}\check{G}_{cc}(\{\chi_\alpha\},\omega)\frac{\partial\check{\Sigma}_c(\{\chi_\alpha\},\omega)}{\partial({\rm i}\chi_{\rm R})}\right\}. \end{aligned} \tag{3.69}$$

This formula describes the cross correlation of current noise in double S/N/N C-SET as far as current continuity condition is taken into account. Using Eq.(3.54), setting the counting field to zero and taking trace in Schwinger-Keldysh space, we arrive at

$$\begin{aligned} S_{\rm LR}(0) =&\frac{e^2}{\hbar^4}|T_{cd_{\rm L}}|^2|T_{cd_{\rm R}}|^2\sum_\sigma\int_{-\infty}^{\infty}\frac{{\rm d}\omega}{4\pi} \\ &\times {\rm Tr}_{NG}\Big[-\Big(\hat{G}^{-+}_{cc}(\omega)\hat{\tau}_3\hat{g}^{+-}_{d_{\rm L1}}(\omega)-\hat{G}^{+-}_{cc}(\omega)\hat{\tau}_3\hat{g}^{-+}_{d_{\rm L1}}(\omega)\Big) \\ &\qquad\times\Big(\hat{G}^{-+}_{cc}(\omega)\hat{\tau}_3\hat{g}^{+-}_{d_{\rm R1}}(\omega)-\hat{G}^{+-}_{cc}(\omega)\hat{\tau}_3\hat{g}^{-+}_{d_{\rm R1}}(\omega)\Big) \\ &-\frac{1}{2}\Big[\Big(\hat{G}^{\rm R}_{cc}(\omega)+\hat{G}^{\rm K}_{cc}(\omega)\Big)\hat{\tau}_3\hat{g}^{-+}_{d_{\rm L1}}(\omega)\hat{G}^{\rm A}_{cc}(\omega)\hat{\tau}_3\hat{g}^{-+}_{d_{\rm R1}}(\omega) \\ &+\hat{G}^{\rm A}_{cc}(\omega)\hat{\tau}_3\hat{g}^{-+}_{d_{\rm L1}}(\omega)\Big(\hat{G}^{\rm R}_{cc}(\omega)-\hat{G}^{\rm K}_{cc}(\omega)\Big)\hat{\tau}_3\hat{g}^{+-}_{d_{\rm R1}}(\omega) \\ &+\Big(\hat{G}^{\rm R}_{cc}(\omega)-\hat{G}^{\rm K}_{cc}(\omega)\Big)\hat{\tau}_3\hat{g}^{+-}_{d_{\rm L1}}(\omega)\hat{G}^{\rm A}_{cc}(\omega)\hat{\tau}_3\hat{g}^{-+}_{d_{\rm R1}}(\omega) \\ &+\hat{G}^{\rm A}_{cc}(\omega)\hat{\tau}_3\hat{g}^{+-}_{d_{\rm L1}}(\omega)\Big(\hat{G}^{\rm R}_{cc}(\omega)+\hat{G}^{\rm K}_{cc}(\omega)\Big)\hat{\tau}_3\hat{g}^{-+}_{d_{i R1}}(\omega)\Big]\Big]. \end{aligned} \tag{3.70}$$

Although the formula Eq.(3.69) and Eq.(3.70) are general, we are interested in the cross correlation of current noise, so that, paying attention on this point and taking trace in Nambu-Gor'kov space, we finally obtain the explicit expression for the *cross correlation in superconducting subgap region*:

$$
\begin{aligned}
S_{\mathrm{LR}}(0) =& \frac{(-e)^2}{2}\Big|\ \frac{1}{\hbar}T_{cd_{\mathrm{L}}}\ \Big|^2\Big|\ \frac{1}{\hbar}T_{cd_{\mathrm{R}}}\ \Big|^2\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\Bigg[\\
& \times\left|\frac{1}{\hbar}T_{cd_{\mathrm{L}}}\right|^2\left|\frac{1}{\hbar}T_{cd_{\mathrm{R}}}\right|^2 \\
& \qquad\times\Big\{|G^{\mathrm{R}}_{cc,11}(\omega)|^4\big(g^{+-}_{d_{\mathrm{L}1},11}(\omega)g^{-+}_{d_{\mathrm{R}1},11}(\omega)-g^{-+}_{d_{\mathrm{L}1},11}(\omega)g^{+-}_{d_{\mathrm{R}1},11}(\omega)\big)^2 \\
& \qquad\quad+|G^{\mathrm{R}}_{cc,22}(\omega)|^4\big(g^{+-}_{d_{\mathrm{L}1},22}(\omega)g^{-+}_{d_{\mathrm{R}1},22}(\omega)-g^{-+}_{d_{\mathrm{L}1},22}(\omega)g^{+-}_{d_{\mathrm{R}1},22}(\omega)\big)^2\Big\} \\
& \quad-\Big\{|G^{\mathrm{R}}_{cc,11}(\omega)|^2\big(g^{+-}_{d_{\mathrm{L}1},11}(\omega)g^{-+}_{d_{\mathrm{R}1},11}(\omega)+g^{-+}_{d_{\mathrm{L}1},11}(\omega)g^{+-}_{d_{\mathrm{R}1},11}(\omega)\big) \\
& \qquad\quad+|G^{\mathrm{R}}_{cc,22}(\omega)|^2\big(g^{+-}_{d_{\mathrm{L}1},22}(\omega)g^{-+}_{d_{\mathrm{R}1},22}(\omega)+g^{-+}_{d_{\mathrm{L}1},22}(\omega)g^{+-}_{d_{\mathrm{R}1},22}(\omega)\big)\Big\} \\
& +\frac{1}{2}\Big(|G^{\mathrm{R}}_{cc,12}(\omega)|^2+|G^{\mathrm{R}}_{cc,21}(\omega)|^2\Big) \\
& \qquad\times\Big(g^{+-}_{d_{\mathrm{L}1},11}(\omega)g^{-+}_{d_{\mathrm{R}1},22}(\omega)+g^{-+}_{d_{\mathrm{L}1},11}(\omega)g^{+-}_{d_{\mathrm{R}1},22}(\omega) \\
& \qquad\quad+g^{+-}_{d_{\mathrm{L}1},22}(\omega)g^{-+}_{d_{\mathrm{R}1},11}(\omega)+g^{-+}_{d_{\mathrm{L}1},22}(\omega)g^{+-}_{d_{\mathrm{R}1},11}(\omega)\Big) \\
& -\Big|\ \frac{1}{\hbar}T_{cd_{\mathrm{L}}}\ \Big|^2\Big|\ \frac{1}{\hbar}T_{cd_{\mathrm{R}}}\ \Big|^2\Big(|G^{\mathrm{R}}_{cc,12}(\omega)|^4+|G^{\mathrm{R}}_{cc,21}(\omega)|^4\Big) \\
& \quad\times\Big\{\big(g^{+-}_{d_{\mathrm{L}1},11}(\omega)g^{-+}_{d_{\mathrm{L}1},22}(\omega)-g^{-+}_{d_{\mathrm{L}1},11}(\omega)g^{+-}_{d_{\mathrm{L}1},22}(\omega)\big) \\
& \qquad\times\big(g^{+-}_{d_{\mathrm{R}1},22}(\omega)g^{-+}_{d_{\mathrm{R}1},11}(\omega)-g^{-+}_{d_{\mathrm{R}1},22}(\omega)g^{+-}_{d_{\mathrm{R}1},11}(\omega)\big) \\
& \qquad+\big(g^{+-}_{d_{\mathrm{L}1},11}(\omega)g^{-+}_{d_{\mathrm{R}1},22}(\omega)-g^{-+}_{d_{\mathrm{L}1},11}(\omega)g^{+-}_{d_{\mathrm{R}1},22}(\omega)\big) \\
& \qquad\quad\times\big(g^{+-}_{d_{\mathrm{R}1},11}(\omega)g^{-+}_{d_{\mathrm{L}1},22}(\omega)-g^{-+}_{d_{\mathrm{R}1},11}(\omega)g^{+-}_{d_{\mathrm{L}1},22}(\omega)\big)\Big\} \\
& -\Big(|G^{\mathrm{R}}_{cc,12}(\omega)|^4+|G^{\mathrm{R}}_{cc,21}(\omega)|^4\Big) \\
& \quad\times\Big(g^{+-}_{d_{\mathrm{L}1},11}(\omega)g^{-+}_{d_{\mathrm{R}1},22}(\omega)-g^{-+}_{d_{\mathrm{L}1},11}(\omega)g^{+-}_{d_{\mathrm{R}1},22}(\omega) \\
& \qquad-g^{+-}_{d_{\mathrm{L}1},22}(\omega)g^{-+}_{d_{\mathrm{R}1},11}(\omega)+g^{-+}_{d_{\mathrm{L}1},22}(\omega)g^{+-}_{d_{\mathrm{R}1},11}(\omega)\Big) \\
& \quad\times\sum_{\alpha}\Big|\ \frac{1}{\hbar}T_{cd_\alpha}\ \Big|^4\big(g^{+-}_{d_{\alpha 1},11}(\omega)g^{-+}_{d_{\alpha 1}22}(\omega)-g^{-+}_{d_{\alpha 1},11}(\omega)g^{+-}_{d_{\alpha 1},22}(\omega)\big)\Bigg]\ .
\end{aligned}
\tag{3.71}
$$

The formula obtained describes non-perturbatively the current noise cross correlation in the double S/N/N C-SET in that the tunneling processes is taken into account up to infinite order.

3.6 Numerical Results

In what follows, for simplicity, we assume the same circuit parameters for left and right C-SETs, *i.e.*, $C_i^{(\alpha)} = C_i \equiv C_0$ $(i = 1, 2)$, $C_3^{(\alpha)} \equiv C_{\mathrm{g}}^{(\alpha)} = C_0/2$ and $L_1^{(\alpha)} = L_2^{(\alpha)} = L$, resulting in $E_{\mathrm{c}}^{(\alpha)} = E_{\mathrm{c}} = e^2/C_0$, $U^{(\alpha)} = U = E_{\mathrm{c}}/5$ and $\omega_j^{(\alpha)}/\omega_{\mathrm{L}}^{(\alpha)} = \omega_j/\omega_{\mathrm{L}}$.

First we consider the case $\mathcal{Q}^{\mathrm{L}} = \mathcal{Q}^{\mathrm{R}} = 1000$ (small Ohmic resistance case). In Fig.3.4 (a)currents, (b)current components and (c)corresponding cross correlations of current noise in subgap region are shown for $V_2^{(\mathrm{R})} = 0.0$, $0.2\cdot(\Delta/e)$ and $0.5\cdot(\Delta/e)$

from top to bottom. Choosing $\Delta/(2U) = 10$ and keeping $V_3^{(\alpha)} = 0$, $V_2^{(\mathrm{L})}$ is changed in the subgap region at temperature $T = 0.2U/k_{\mathrm{B}}$ and $R_{\mathrm{T}}^{(\alpha)}/R_{\mathrm{q}} = 100$ (tunneling limit). With this parameter choice, there are three kinds of currents, $I_\alpha^{(\mathrm{cAR})}$, $I_\alpha^{(\mathrm{dAR})}$, and $I_\alpha^{(\mathrm{EC})}$ in subgap region, but note that the contribution to $S_{\mathrm{LR}}(\omega = 0)$ comes from $I_\alpha^{(\mathrm{cAR})}$ and $I_\alpha^{(\mathrm{EC})}$, yielding bunching and anti-bunching correlations, respectively. Since $I_\alpha^{(\mathrm{cAR})}$ ($I_\alpha^{(\mathrm{EC})}$) becomes zero at $V_{\mathrm{I}}^{(+)} \equiv V_{\mathrm{I}}^{(\mathrm{L})} + V_{\mathrm{I}}^{(\mathrm{R})} = 0$ ($V_{\mathrm{I}}^{(-)} \equiv V_{\mathrm{I}}^{(\mathrm{L})} - V_{\mathrm{I}}^{(\mathrm{R})} = 0$) in the absence of charging energy, Coulomb gap for $I_\alpha^{(\mathrm{cAR})}$ ($I_\alpha^{(\mathrm{EC})}$) opens in the region $|V_{\mathrm{I}}^{(+)}| < 2U/e$ ($|V_{\mathrm{I}}^{(-)}| < 2U/e$). Note that chemical potential of island in α C-SET, $V_{\mathrm{I}}^{(\alpha)}$ can be obtained self-consistently by imposing the current continuity condition of the double S/N/N C-SET and is increasing function of $V_2^{(\alpha)}$.

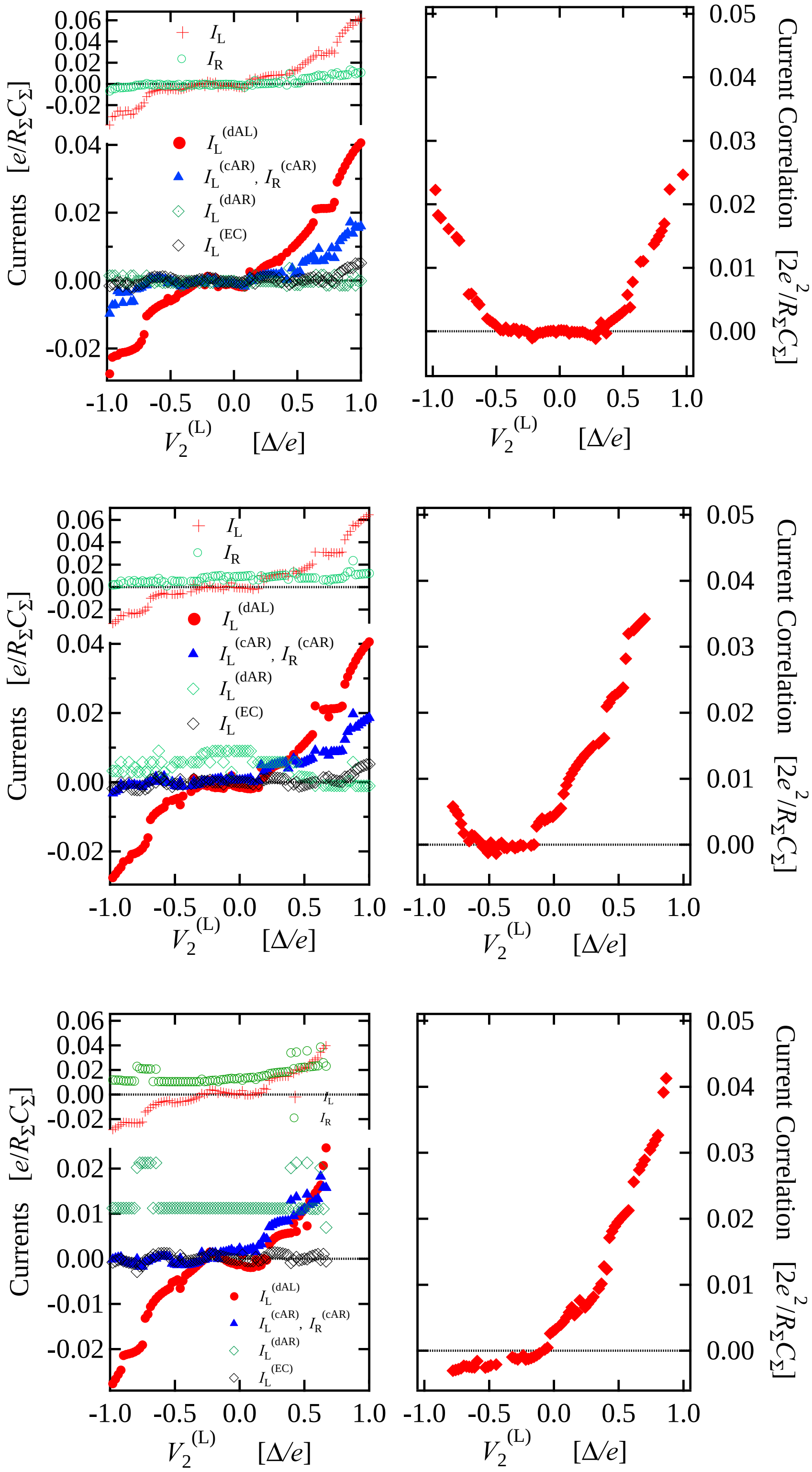


Fig. 3.4 For double S/N/N C-SET with $\mathcal{Q}^{(\alpha)} = 1000$, $V_1 = 0$, and $V_3^{(\alpha)} = 0$, currents I_α, current components due to direct (crossed) Andreev reflection $I_\alpha^{(\mathrm{dAR})}$ ($I_\alpha^{(\mathrm{cAR})}$), the elastic co-tunneling $I_\alpha^{(\mathrm{EC})}$ flowing through α (=L, R) C-SET and corresponding current noise cross correlation $S_{\mathrm{LR}}(0)$ are shown as a function of bias voltage $V_2^{(\mathrm{L})}$ with $V_2^{(\mathrm{R})} = 0$ (top), $V_2^{(\mathrm{R})} = 0.2 \cdot \Delta/e$ (middle), and $V_2^{(\mathrm{R})} = 0.5 \cdot \Delta/e$ (bottom). Temperature is tentatively chosen as $T = 0.2U/k_{\mathrm{B}}$.

In the case $V_2^{(\mathrm{R})} = 0.0$, $S_{\mathrm{LR}}(\omega = 0)$ is positive definite, *i.e.*, the bunching correlation in whole subgap region since $|I_\alpha^{(\mathrm{cAR})}|$ is always larger than $|I_\alpha^{(\mathrm{EC})}|$ for non-zero $V_{\mathrm{I}}^{(\mathrm{L})}$, and both currents are zero at $V_2^{(\mathrm{L})} = 0$. Therefore $S_{\mathrm{LR}}(\omega = 0)$ in this case is symmetric function with respect to $V_2^{(\mathrm{L})} = 0$ axis in subgap region.

In the case $V_2^{(\mathrm{R})} = 0.2 \cdot \Delta/e$, on the other hand, Coulomb gap regions for $I_\alpha^{(\mathrm{cAR})}$ and $I_\alpha^{(\mathrm{EC})}$ do not coincide. Because of that $S_{\mathrm{LR}}(\omega = 0)$ becomes negative (*typical noise correlation for particles which obey Fermi-Dirac statistics*) in a narrow window of $V_2^{(\mathrm{L})}$ with center at around $V_{\mathrm{I}}^{(+)} = 0$ where $I_\alpha^{(\mathrm{cAR})} = 0$ while $I_\alpha^{(\mathrm{EC})}$ is non-zero (*bunching-antibunching crossover*), and becomes asymmetric with respect to $V_2^{(\mathrm{L})} = 0$ axis. For this value of $V_2^{(\mathrm{R})}$, however, $I_\alpha^{(\mathrm{cAR})}$ rapidly becomes more dominant than $I_\alpha^{(\mathrm{EC})}$ as $V_2^{(\mathrm{L})}$ increases beyond Coulomb gap region of $I_\alpha^{(\mathrm{cAR})}$. As a result, $S_{\mathrm{LR}}(\omega = 0)$ becomes positive again (*Restoration of bunching correlation*). Therefore, we can expect the bunching correlation of current noise over a wide range of $V_2^{(\mathrm{L})}$ even in this case.

Furthermore, in the case $V_2^{(\mathrm{R})} = 0.5 \cdot \Delta/e$, although the situation is similar to the case $V_2^{(\mathrm{R})} = 0.2 \cdot \Delta/e$ in that Coulomb gap regions for $I_\alpha^{(\mathrm{cAR})}$ and $I_\alpha^{(\mathrm{EC})}$ do not coincide, distance between two Coulomb gap regions along $V_2^{(\mathrm{L})} = 0$ axis becomes much larger. Because of that, $I_\alpha^{(\mathrm{cAR})}$ becomes less dominant than $I_\alpha^{(\mathrm{EC})}$ in superconductor subgap region ($V_2^{(\mathrm{L})} > \Delta/e$) beyond Coulomb gap region of $I_\alpha^{(\mathrm{cAR})}$. As a result, $S_{\mathrm{LR}}(\omega = 0)$ never becomes positive (*bunching-antibunching crossover and luck of restoration of bunching*).

In Fig.3.5 cross correlation of current noise $S_{\mathrm{LR}}(0)$ are shown for $\mathcal{Q}^{(\alpha)} = 1000$ (low Ohmic resistance) and $\mathcal{Q}^{(\alpha)} = 0.034$ (high ohmic resistance). Results show that bunching-antibunching region becomes wider partly because Coulomb gap regions for $I_\alpha^{(\mathrm{cAR})}$ becomes larger and partly because de-coherence due to Ohmic dissipation makes $I_\alpha^{(\mathrm{cAR})}$ which conveys quantum entangled information smaller in principle.

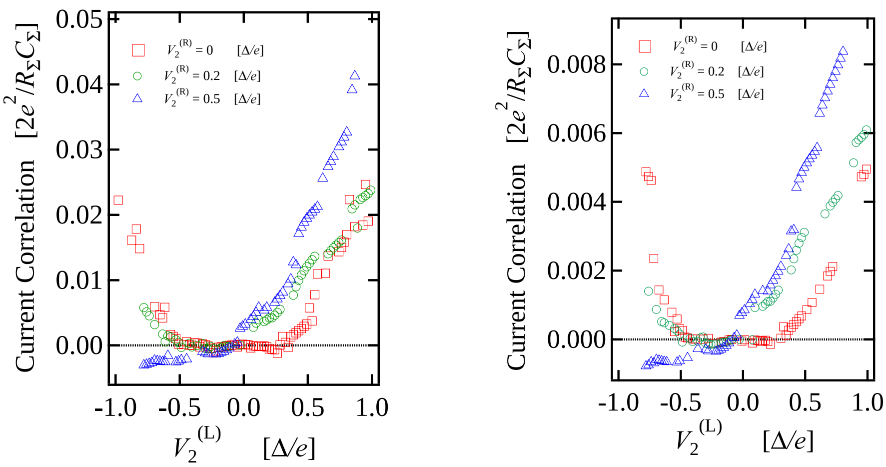


Fig. 3.5 Cross correlation of current noise: $S_{\mathrm{LR}}(0)$ for $Q = 1000$ (left) and $S_{\mathrm{LR}}(0)$ for $\mathcal{Q}^{(\alpha)} = 0.034$ (right)

Chapter 4

CONTROL OF CROSS CORRELATION BY MAGNETIC ORDER

4.1 Ferromagnetic Island: Double S/F/N C-SET

4.1.1 Model and Hamiltonian

In the preceding chapter we have discussed current and cross correlation of current noise of the double S/N/N C-SET in the superconducting subgap region and have shown that the crossed Andreev current conveys non-local spin entangled information and causes the bunching correlation. In this situation, however, current due to elastic co-tunneling also exists and causes the antibunching correlation peculiar to fermion flow.

Let us consider here a kind of filter for the local spin entangled information. One of the candidates is the SSE whose structure is the same structure as S/N/N C-SET but with nonmagnetic islands replaced by ferromagnetic islands (Fig.4.1). In what follows we call this SSE as *double S/F/N C-SET*.

Hamiltonian of the double S/F/N C-SET is given as

$$K_{\rm es} = \frac{1}{2}\sum_{\boldsymbol{k}\sigma} \hat{\Psi}^{(c)\dagger}_{\boldsymbol{k}\sigma} \Big\{ \xi^{(c)}_{\boldsymbol{k}} \hat{\tau}_3 - \frac{\sigma}{2}\big[\Delta\hat{\tau}_+ - \Delta^*\hat{\tau}_-\big] \Big\} \hat{\Psi}^{(c)}_{\boldsymbol{k}'\sigma} + \frac{1}{2}\sum_{\alpha}\sum_{\boldsymbol{k}\sigma} \hat{\Psi}^{(a_\alpha)\dagger}_{\boldsymbol{k}\sigma}\ \xi^{(a_\alpha)}_{\boldsymbol{k}}\ \hat{\tau}_3\ \hat{\Psi}^{(a_\alpha)}_{\boldsymbol{k}'\sigma}$$
$$+\frac{1}{2}\sum_{\alpha}\sum_{\boldsymbol{k}\sigma} \hat{\Psi}^{(d_\alpha)\dagger}_{\boldsymbol{k}\sigma} \left(\xi^{(d_\alpha)}_{\boldsymbol{k}} - \gamma_\alpha \sigma \frac{\Delta^{(\alpha)}_{\rm ex}}{2} \right) \hat{\tau}_3\ \hat{\Psi}^{(d_\alpha)}_{\boldsymbol{k}'\sigma}\ , \tag{4.1}$$

where $\Delta^{(\alpha)}_{\rm ex}$ is the exchange splitting energy of α ferromagnetic island and γ_α denotes the spin alignment of α ferromagnetic island. $\gamma_\alpha = +1$ ($\gamma_\alpha = -1$) corresponds to majority spin in α island is up (down).

4.1.2 Effect of magnetic ordering

In the presence of ferromagnetic ordering the density of states at the Fermi energy is not equal for the spin orientations. Since tunneling process never change the spin orientation as far as magnetic impurities are absent, tunneling probability is strongly affected depending on whether the tunneling electron has majority spin or minority spin.

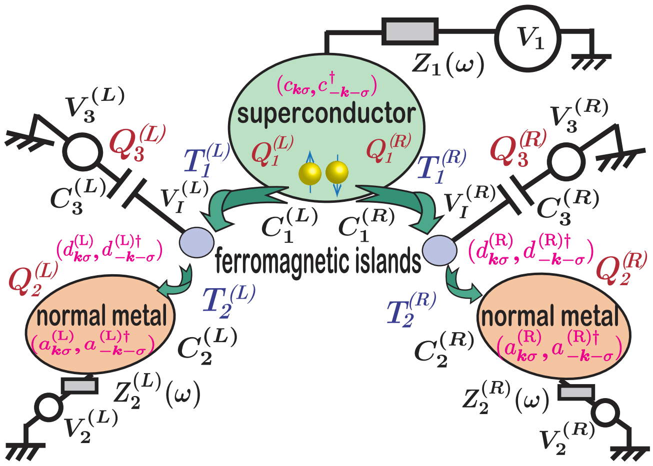


Fig. 4.1 The structure of double S/F/N C-SET with external circuit. Metallic islands are assumed ferromagnetic. Magnetic ordering serves a kind of filter for non-local spin entangled information.

The lesser and greater functions for the electrons in the ferromagnetic island of α C-SET are given in Nambu-Gor'kov space as follows:

$$
\begin{aligned}
g^{+-}_{d_{\alpha i}}(\omega) &= -\mathrm{i}\pi\hbar\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega'}{2\pi}(1-n_{\mathrm{F}}(\epsilon-\epsilon'))\\
&\quad\times\left\{\begin{array}{cc} N_{d_\alpha}(0)(1+\gamma_\alpha\sigma P_\alpha)\mathcal{F}^{(\alpha i)+-}_{+-}(\omega') & 0\\ 0 & N_{d_\alpha}(0)(1-\gamma_\alpha\sigma P_\alpha)\mathcal{F}^{(\alpha i)+-}_{-+}(\omega')\end{array}\right\},\\
g^{-+}_{d_{\alpha i}}(\omega) &= \mathrm{i}\pi\hbar\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega'}{2\pi}n_{\mathrm{F}}(\epsilon-\epsilon')\\
&\quad\times\left\{\begin{array}{cc} N_{d_\alpha}(0)(1+\gamma_\alpha\sigma P_\alpha)\mathcal{F}^{(\alpha i)-+}_{+-}(\omega') & 0\\ 0 & N_{d_\alpha}(0)(1-\gamma_\alpha\sigma P_\alpha)\mathcal{F}^{(\alpha i)-+}_{-+}(\omega')\end{array}\right\}.
\end{aligned}
\tag{4.2}
$$

Here $\epsilon=\hbar\omega$, and $n_{\mathrm{F}}(\epsilon)=[\exp(\beta\epsilon)+1]^{-1}$ and $N_{d_\alpha}(0)$ are, respectively, Fermi distribution function and the density of states at Fermi energy for the metal of α island in nonmagnetic state. $\mathcal{F}^{(\alpha i)}_{\lambda\lambda'}(\omega)=\mathcal{F}^{(\alpha i)}_{\lambda\lambda',\mathrm{env}}(\omega)\mathcal{F}^{(\alpha i)}_{\lambda\lambda',\mathrm{c}}(\omega)$ is the product of correlation functions of environmental and charging phases. P_α denotes the spin polarization rate, and how to enlarge it is one of the most interested problems in application-oriented research fields such as *spintronics*.

4.2 Current and Cross Correlation of Current Noise

The current flowing through the double S/F/N C-SET in subgap region ($e(V_{\rm I}^{(\alpha)} - V_1) < \Delta$) is obtained as

$$
\begin{aligned}
I_\alpha =& \frac{e}{2}\int_{-\infty}^{\infty}\frac{{\rm d}\omega}{2\pi}\Bigg[\left|\frac{1}{\hbar}T_{cd_\alpha}\right|^2 \\
&\times\Bigg\{\left|\frac{1}{\hbar}T_{cd_{\bar\alpha}}\right|^2\left|G^{\rm R}_{cc,11}(\omega)\right|^2\Big(g^{+-}_{d_{\alpha1},11}(\omega)g^{-+}_{d_{\bar\alpha1},11}(\omega)-g^{-+}_{d_{\alpha1},11}(\omega)g^{+-}_{d_{\bar\alpha1},11}(\omega)\Big) \\
&+\sum_{\alpha'={\rm L,R}}\left|\frac{1}{\hbar}T_{cd_\alpha}\right|^2\left|G^{\rm R}_{cc,12}(\omega)\right|^2\Big(g^{+-}_{d_{\alpha1},11}(\omega)g^{-+}_{d_{\alpha'1},22}(\omega)-g^{-+}_{d_{\alpha1},11}(\omega)g^{+-}_{d_{\alpha'1},22}(\omega)\Big)\Bigg\} \\
&-\left|\frac{1}{\hbar}T_{a_\alpha d_\alpha}\right|^2\left|\;1+T_{d_\alpha a_\alpha}G^{\rm R}_{cd_\alpha,11}(\omega)\;\right|^2 \\
&\qquad\times\Big(g^{+-}_{d_{\alpha2},11}(\omega)g^{-+}_{a_\alpha,11}(\omega)-g^{-+}_{d_{\alpha2},11}(\omega)g^{+-}_{a_\alpha,11}(\omega)\Big)\Bigg], \qquad (4.3)
\end{aligned}
$$

together with current continuity condition. This current includes currents only due to EC, cAR, dAR.

The product of the lesser and greater Green's functions in the same island is given by

$$g^{+-}_{d_{\alpha1},11}(\omega)g^{-+}_{d_{\alpha1},22}(\omega)\propto N_{d_\alpha}(0)N_{d_\alpha}(0)(1+\sigma P_\alpha)(1-\sigma P_\alpha). \qquad (4.4)$$

This term describes the current due to dAR, $I_\alpha^{(dAR)}$ and is suppressed in ferromagnetic island. In the limit of half metal, in which all the electrons have the same spin polarization ($P_\alpha = 1$), $I_\alpha^{(dAR)}$ disappears irrespective of spin align configuration of ferromagnetic islands. Therefore, in subgap region only currents due to EC and cAR processes, $I_\alpha^{(EC)}$ and $I_\alpha^{(cAR)}$ exist. In what follows we restrict ourselves to half metal islands only.

4.2.1 Parallel configuration

Let us consider the parallel configuration. In this case the majority spins of both ferromagnetic islands are the same; if $\gamma_{\rm L} = \gamma_{\rm R} = +1$, both majority spins are up. For the current due to cAR reflection, the product of the lesser and greater Green's functions becomes

$$g^{+-}_{d_{\rm L1},11}(\omega)g^{-+}_{d_{\rm R1},22}(\omega)\propto N_{d_{\rm L}}(0)N_{d_{\rm R}}(0)(1+\sigma P_{\rm L})(1-\sigma P_{\rm R})\,, \qquad (4.5)$$

which vanishes in half metal limit ($P_{\rm L} = P_{\rm R} = 1$). Therefore the current in parallel alignment, $I_{\alpha,\parallel}$ for half metal islands originates from EC only, and is given as

$$
\begin{aligned}
I_{\alpha,\parallel} =&4\pi^2 e \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega_1}{2\pi} \\
&\times \Bigg[\int_{-\infty}^{\infty} \frac{\mathrm{d}\omega_2}{2\pi} \, 4\Big|\frac{1}{\hbar}T_{cd_\mathrm{L}}\Big|^2 \Big|\frac{1}{\hbar}T_{cd_\mathrm{R}}\Big|^2 \left|G^\mathrm{R}_{cc,11}(\omega)\right|^2 N_{d_\mathrm{L}}(0) N_{d_\mathrm{R}}(0) \\
&\quad \times \Big\{ \mathcal{F}^{(\mathrm{L1})+-}_{+-}(\hbar\omega_1)\big[1 - n_\mathrm{F}\big(\hbar(\omega-\omega_1) + e(V^{(\mathrm{L})}_1 - V^{(\mathrm{L})}_I)\big)\big] \\
&\qquad \times \mathcal{F}^{(\mathrm{R1})-+}_{+-}(\hbar\omega_2) n_\mathrm{F}\big(\hbar(\omega-\omega_2) + e(V^{(\mathrm{R})}_1 - V^{(\mathrm{R})}_I)\big) \\
&\quad - \mathcal{F}^{(\mathrm{L1})-+}_{+-}(\hbar\omega_1) n_\mathrm{F}\big(\hbar(\omega-\omega_1) + e(V^{(\mathrm{L})}_1 - V^{(\mathrm{L})}_I)\big) \\
&\qquad \times \mathcal{F}^{(\mathrm{R1})+-}_{+-}(\hbar\omega_2)\big[1 - n_\mathrm{F}\big(\hbar(\omega-\omega_2) + e(V^{(\mathrm{R})}_1 - V^{(\mathrm{R})}_I)\big)\big] \Big\} \\
&- 2\Big|\frac{1}{\hbar}T_\alpha 2\Big|^2 \Big| \, 1 + T_{\alpha *} 2 G^\mathrm{R}_{cd_\alpha,11}(\omega) \, \Big|^2 N_{d_\alpha}(0) N_{a_\alpha}(0) \\
&\quad \times \mathcal{F}^{(\mathrm{L2})+-}_{+-}(\hbar\omega_1)\big[1 - n_\mathrm{F}\big(\hbar(\omega-\omega_1) + e(V^{(\alpha)}_2 - V^{(\alpha)}_I)\big)\big] n_\mathrm{F}(\hbar\omega) \\
&\qquad - \mathcal{F}^{(\mathrm{L2})-+}_{+-}(\hbar\omega_1) n_\mathrm{F}\big(\hbar(\omega-\omega_1) + e(V^{(\alpha)}_2 - V^{(\alpha)}_I)\big)\big[1 - n_\mathrm{F}(\hbar\omega)\big] \Bigg] . \qquad (4.6)
\end{aligned}
$$

Because of the similar reason, there are also vanishing terms in cross correlation of current noise in subgap region (Eq.(3.71)). Therefore, in the case of parallel alignment for half metal islands, cross correlation of current noise is given as

$$
\begin{aligned}
S_{\mathrm{LR},\parallel}(0) =&\frac{(-e)^2}{2} \Big| \, \frac{1}{\hbar}T_{cd_\mathrm{L}} \, \Big|^2 \Big| \, \frac{1}{\hbar}T_{cd_\mathrm{R}} \, \Big|^2 \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \\
&\times \Bigg[\Big|\frac{1}{\hbar}T_{cd_\mathrm{L}}\Big|^2 \Big|\frac{1}{\hbar}T_{cd_\mathrm{R}}\Big|^2 \\
&\qquad \times \Big\{ |G^\mathrm{R}_{cc,11}(\omega)|^4 \big(g^{+-}_{d_\mathrm{L1},11}(\omega) g^{-+}_{d_\mathrm{R1},11}(\omega) - g^{-+}_{d_\mathrm{L1},11}(\omega) g^{+-}_{d_\mathrm{R1},11}(\omega)\big)^2 \\
&\qquad\quad + |G^\mathrm{R}_{cc,22}(\omega)|^4 \big(g^{+-}_{d_\mathrm{L1},22}(\omega) g^{-+}_{d_\mathrm{R1},22}(\omega) - g^{-+}_{d_\mathrm{L1},22}(\omega) g^{+-}_{d_\mathrm{R1},22}(\omega)\big)^2 \Big\} \\
&\quad - \Big\{ |G^\mathrm{R}_{cc,11}(\omega)|^2 \big(g^{+-}_{d_\mathrm{L1},11}(\omega) g^{-+}_{d_\mathrm{R1},11}(\omega) + g^{-+}_{d_\mathrm{L1},11}(\omega) g^{+-}_{d_\mathrm{R1},11}(\omega)\big) \\
&\qquad\quad + |G^\mathrm{R}_{cc,22}(\omega)|^2 \big(g^{+-}_{d_\mathrm{L1},22}(\omega) g^{-+}_{d_\mathrm{R1},22}(\omega) + g^{-+}_{d_\mathrm{L1},22}(\omega) g^{+-}_{d_\mathrm{R1},22}(\omega)\big) \Big\} \Bigg] , \qquad (4.7)
\end{aligned}
$$

under current continuity condition.

4.2.2 Anti-parallel configuration

Next let us consider the anti-parallel configuration. In this case the majority spins of both ferromagnetic islands are opposite; if $\gamma_\mathrm{L} = +1$ and $\gamma_\mathrm{R} = -1$, majority spins of electrons in left and right islands are, respectively up and down.

For the current due to EC, the product of the lesser and greater Green's functions of electron becomes

$$
g^{+-}_{d_\mathrm{L1},11}(\omega) g^{-+}_{d_\mathrm{R1},11}(\omega) \propto N_{d_\mathrm{L}}(0) N_{d_\mathrm{R}}(0)(1 + \sigma P_\mathrm{L})(1 - \sigma P_\mathrm{R}) . \qquad (4.8)
$$

Therefore, in this configuration, the current due to EC vanishes in half metal limit.

The current in anti-parallel alignment, $I_{\alpha,\perp}$ for half metal islands is given as

$$\begin{aligned}
I_{\alpha,\perp} =& 2\pi^2 e \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega_1}{2\pi} \\
& \times \Bigg[\int_{-\infty}^{\infty} \frac{\mathrm{d}\omega_2}{2\pi}\, 4 \Big|\frac{1}{\hbar} T_{cd_{\mathrm{L}}}\Big|^2 \Big|\frac{1}{\hbar} T_{cd_{\mathrm{R}}}\Big|^2 \left|G^{\mathrm{R}}_{cc,12}(\omega)\right|^2 N_{d_{\mathrm{L}}}(0) N_{d_{\mathrm{R}}}(0) \\
& \quad \times \Big\{ \mathcal{F}^{(\mathrm{L1})+-}_{+-}(\hbar\omega_1) \big[1 - n_{\mathrm{F}}\big(\hbar(\omega-\omega_1) + e(V_1^{(\mathrm{L})} - V_I^{(\mathrm{L})})\big)\big] \\
& \qquad \times \mathcal{F}^{(\mathrm{R1})-+}_{-+}(\hbar\omega_2) n_{\mathrm{F}}\big(\hbar(\omega-\omega_2) - e(V_1^{(\mathrm{R})} - V_I^{(\mathrm{R})})\big) \\
& \quad - \mathcal{F}^{(\mathrm{L1})-+}_{+-}(\hbar\omega_1) n_{\mathrm{F}}\big(\hbar(\omega-\omega_1) + e(V_1^{(\mathrm{L})} - V_I^{(\mathrm{L})})\big) \\
& \qquad \times \mathcal{F}^{(\mathrm{R1})+-}_{-+}(\hbar\omega_2) \big[1 - n_{\mathrm{F}}\big(\hbar(\omega-\omega_2) - e(V_1^{(\mathrm{R})} - V_I^{(\mathrm{R})})\big)\big] \Big\} \\
& - 2 \Big|\frac{1}{\hbar} T_{\alpha} 2\Big|^2 \Big| 1 + T_{\alpha *} 2 G^{\mathrm{R}}_{cd_\alpha,11}(\omega) \Big|^2 N_{d_\alpha}(0) N_{a_\alpha}(0) \\
& \quad \times \mathcal{F}^{(\mathrm{L2})+-}_{+-}(\hbar\omega_1) \big[1 - n_{\mathrm{F}}\big(\hbar(\omega-\omega_1) + e(V_2^{(\alpha)} - V_I^{(\alpha)})\big)\big] n_{\mathrm{F}}(\hbar\omega) \\
& \qquad - \mathcal{F}^{(\mathrm{L2})-+}_{+-}(\hbar\omega_1) n_{\mathrm{F}}\big(\hbar(\omega-\omega_1) + e(V_2^{(\alpha)} - V_I^{(\alpha)})\big) \big[1 - n_{\mathrm{F}}(\hbar\omega)\big] \Bigg]. \quad (4.9)
\end{aligned}$$

The current in Eq.(4.9) contains only cAR process between superconductor/half-metal-islands interface, so we can extract the cAR current which includes quantum entanglement information. Therefore the introduction of ferromagnetic islands enables us to extract and control the quantum entanglement.

The cross correlation of current noise in subgap region (Eq.(3.71)) in anti-parallel alignment for half metal islands is obtained as

$$\begin{aligned}
S_{\mathrm{LR},\perp}(0) =& \frac{(-e)^2}{2} \Big| \frac{1}{\hbar} T_{cd_{\mathrm{L}}} \Big|^2 \Big| \frac{1}{\hbar} T_{cd_{\mathrm{R}}} \Big|^2 \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \Bigg[\frac{1}{2} \Big(|G^{\mathrm{R}}_{cc,12}(\omega)|^2 + |G^{\mathrm{R}}_{cc,21}(\omega)|^2 \Big) \\
& \times \Big(g^{+-}_{d_{\mathrm{L1}},11}(\omega) g^{-+}_{d_{\mathrm{R1}},22}(\omega) + g^{-+}_{d_{\mathrm{L1}},11}(\omega) g^{+-}_{d_{\mathrm{R1}},22}(\omega) \\
& \quad + g^{+-}_{d_{\mathrm{L1}},22}(\omega) g^{-+}_{d_{\mathrm{R1}},11}(\omega) + g^{-+}_{d_{\mathrm{L1}},22}(\omega) g^{+-}_{d_{\mathrm{R1}},11}(\omega) \Big) \\
& - \Big| \frac{1}{\hbar} T_{cd_{\mathrm{L}}} \Big|^2 \Big| \frac{1}{\hbar} T_{cd_{\mathrm{R}}} \Big|^2 \Big(|G^{\mathrm{R}}_{cc,12}(\omega)|^4 + |G^{\mathrm{R}}_{cc,21}(\omega)|^4 \Big) \\
& \times \Big\{ \big(g^{+-}_{d_{\mathrm{L1}},11}(\omega) g^{-+}_{d_{\mathrm{L1}},22}(\omega) - g^{-+}_{d_{\mathrm{L1}},11}(\omega) g^{+-}_{d_{\mathrm{L1}},22}(\omega) \big) \\
& \quad \times \big(g^{+-}_{d_{\mathrm{R1}},22}(\omega) g^{-+}_{d_{\mathrm{R1}},11}(\omega) - g^{-+}_{d_{\mathrm{R1}},22}(\omega) g^{+-}_{d_{\mathrm{R1}},11}(\omega) \big) \\
& \quad + \big(g^{+-}_{d_{\mathrm{L1}},11}(\omega) g^{-+}_{d_{\mathrm{R1}},22}(\omega) - g^{-+}_{d_{\mathrm{L1}},11}(\omega) g^{+-}_{d_{\mathrm{R1}},22}(\omega) \big) \\
& \quad \times \big(g^{+-}_{d_{\mathrm{R1}},11}(\omega) g^{-+}_{d_{\mathrm{L1}},22}(\omega) - g^{-+}_{d_{\mathrm{R1}},11}(\omega) g^{+-}_{d_{\mathrm{L1}},22}(\omega) \big) \Big\} \Bigg]. \quad (4.10)
\end{aligned}$$

Fig.4.2 shows Feynman diagrams of crossed Andreev reflection processes. In half metal limit, process (b) is forbidden because both electrons of up and down spins are required in the one island.

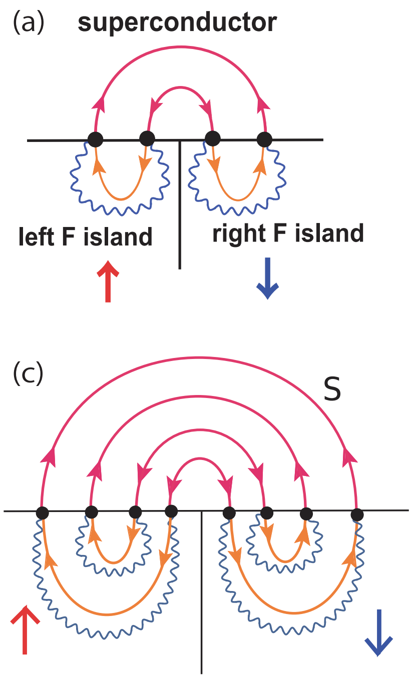


Fig. 4.2 Feynman diagrams describing transport of crossed Andreev reflection in anti-parallel configuration, where solid lines of orange (magenta) are the lowest Green's functions of island electrons (superconductor electrons) and blue wavy lines are correlation functions of phases. Process (b) is a forbidden one in half metal limit.

Although the anti-parallel configuration of island spins is promising to purely extracts quantum entanglement information, the bunching signal becomes a little smaller since some class of higher order of tunneling processes are quenched.

Chapter 5

SUMMARY

We proposed the non-perturbative theory for full counting statistics (FCS) in solid state entangler (SSE) based on Nambu-Gor'kov and Schwinger-Keldysh field theoretical non-equilibrium method. Based on the theory we studied the currents and cross correlation of the current noise of SSE to get the further understanding of physics of quantum entanglement. The theory can be applicable to the transport properties in the region where Coulomb blockade and Andreev reflection coexist, since tunneling processes are taken into account non-perturbatively in the presence of arbitrarily large charging effect as well as electromagnetic environment effect. Concerning the latter, we treated SSE with ohmic resistance phenomenologically in the spirit of Cardeira-Leggett theory.

We considered *double S/N/N C-SET* as SSE, the structure of which consists of a common superconducting electrode/normalconducting left and right ultra-small central electrodes (islands)/normalconducting left and right drains. Each of islands is capacitively coupled with gate electrodes to control tunneling. Since we were interested in extracting and controlling quantum entanglement information by the charging effect, we studied SSE with $U < \Delta$ (U: charging energy, Δ: energy gap of superconductor) so that sufficiently wide superconducting subgap region can be expected.

We derived explicit expression for cumulant generation function (CGF) for the double S/N/N C-SET. Explicit expressions for the currents and cross correlation of current noise were then obtained as the first and second derivatives (cumulants) with respect to the relevant counting fields, respectively. It was shown that, in the subgap region there exist three kinds of currents due to the direct Andreev reflection (dAR), crossed Andreev reflection (cAR) and elastic co-tunneling (EC) and each of the currents exhibit Coulomb blockade related phenomena (Coulomb gaps, Coulomb staircases and Coulomb oscillations) due to the charging effect. Since contribution to the cross correlation of current noise $S_{\mathrm{LR}}(0)$ comes from currents due to cAR and EC in subgap region, the correlation is strongly influenced by the charging effect.

We showed that $S_{\mathrm{LR}}(0)$ is always positive (bunching correlation) in the subgap region if two C-SETs are biased symmetrically. The cross correlation for the flow of Fermi particles is antibunching because of Fermi-Dirac statistics. Therefore, it is the direct consequence of the fermion flow with quantum entanglement. It was also shown that $S_{\mathrm{LR}}(0)$ exhibit *bunching-antibunching crossover* followed by *restoration of bunching correlation* as bias voltage increases and $S_{\mathrm{LR}}(0)$ shows *bunching-antibunching crossover* only and never shows the restoration of bunching correlation for further large asymmetric bias condition.

Effect of ohmic resistance on $S_{\mathrm{LR}}(0)$ was studied. The ohmic dissipation strongly influences on $S_{\mathrm{LR}}(0)$, partly because of increase in Coulomb Gap of cAR current, and partly because of decrease in magnitude of cAR current.

We proposed *double S/F/N C-SET*, structure of which is the double S/N/N C-SET but with ferromagnetic islands instead of nonmagnetic ones as one of the methods for extraction and control of quantum entanglement information. It was shown that if the half metal ferromagnetic islands are employed, only cAR current (EC current) contributes to $S_{\mathrm{LR}}(0)$; *i.e.*, perfect extraction of bunching and antibunching correlation.

Theoretical results obtained in this study were all made for the first time in this field, and the results will lead us to the new stage of the study on bunching-antibunching nature of current noise cross correlation in the presence of quantum entanglement.

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Appendix A

C-SET Theory

A.1 Describing electromagnetic environment effect and charging effect in C-SET

Basic structure to investigate Coulomb blockade is C-SET. d In the microscopic C-SET theory [5, 41–45, 48] charges induced on the electrodes are all treated as quantum mechanical macroscopic variables. First thing to do is to find phases which are canonical conjugate to the charges. It can be made based on *quantum mechanics with constraint* by Dirac [49]. In this case, the constraint condition is that the charge of island

$$q \equiv -\sum_{i=1}^{3} Q_i, \tag{A.1}$$

is constant in time in the absence of tunneling:

$$\sum_{i=1}^{3} \dot{Q}_i = 0. \tag{A.2}$$

Lagrangian of the electromagnetic energy $\mathcal{L}_{\rm em}$ for C-SET structure is given by

$$\mathcal{L}_{\rm em} = \sum_{i=1}^{2} \left\{ \frac{L_i}{2}\dot{Q}_i^2 - \frac{Q_i^2}{2C_i} + Q_i(V_i - V_c) \right\} - \frac{Q_3^2}{2C_3} + Q_3(V_3 - V_c) + \lambda \sum_{i=1}^{3} \dot{Q}_i. \tag{A.3}$$

The canonically conjugate phase variables are determined by

$$\varphi_i = \frac{\partial \mathcal{L}_{\rm em}}{\partial \dot{Q}_i} = L_i \dot{Q}_i + \lambda \quad (i = 1, 2), \tag{A.4}$$

$$\varphi_3 = \frac{\partial \mathcal{L}_{\rm em}}{\partial \dot{Q}_3} = \lambda, \tag{A.5}$$

$$\pi = \frac{\partial \mathcal{L}_{\rm em}}{\partial \dot{\lambda}} = 0. \tag{A.6}$$

Therefore resulting Hamiltonian is form of

$$\begin{aligned} \mathcal{H}_{\rm em} &= \sum_{i=1}^{3} \varphi_i \dot{Q}_i + \pi \dot{\lambda} - \mathcal{L}_{\rm em} \\ &= \sum_{i=1}^{2} \left\{ \frac{1}{2L_i}(\varphi_i - \lambda)^2 + \frac{Q_i^2}{2C_i} - Q_i(V_i - V_c) \right\} + \frac{Q_3^2}{2C_3} - Q_3(V_3 - V_c). \end{aligned} \tag{A.7}$$

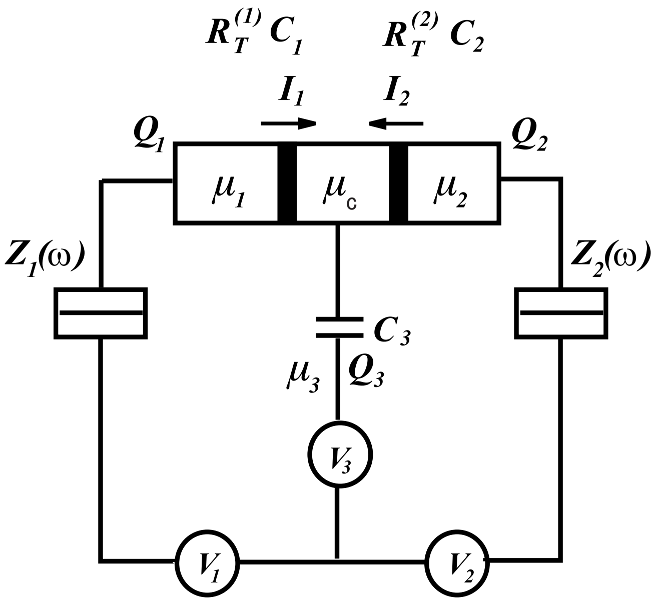

Fig. A.1 The ultra-small double junction with island tuned by gate voltage (C-SET). $R_{\mathrm{T}}^{(j)}$ and C_j are tunnel resistance and capacitance of the junction $j(j = 1, 2)$. μ_i is chemical potential of the electrode i (left electrode $i = 1$, right electrode $i = 2$, gate electrode $i = 3$, island $i = c$), Q_i is the charge of the electrode i and V_i is the voltage expressed as $V_i - V_c = -(\mu_i - \mu_c)/e$ of the electrode i.

Defining $\chi_1 \equiv \lambda - \varphi_3 = 0$ and $\chi_2 \equiv \pi = 0$, Hamiltonian is enable to be rewritten as

$$\mathcal{H}'_{\mathrm{em}} \equiv \mathcal{H}_{\mathrm{em}} + \sum_{i=1}^{2} \alpha_i \chi_i. \tag{A.8}$$

Noting that $[\chi_1, \chi_2] = \mathrm{i}\hbar$, the equation of motion for χ_1 is given by

$$\begin{aligned} \mathrm{i}\hbar\dot{\chi_1} &= [\chi_1, \mathcal{H}'_{\mathrm{em}}] = [\chi_1, \mathcal{H}_{\mathrm{em}}] + \alpha_2[\chi_1, \chi_2] \\ &= \mathrm{i}\hbar\left[\frac{Q_3}{C_3} - (V_3 - V_c) + \alpha_2\right] = 0, \end{aligned} \tag{A.9}$$

therefore α_2 is given by

$$\alpha_2 = -\left[\frac{Q_3}{C_3} - (V_3 - V_c)\right]. \tag{A.10}$$

Similarly the equation of motion for χ_2 is given by

$$\mathrm{i}\hbar\dot{\chi_2} = \mathrm{i}\hbar\left[\sum_{i=1}^{2} \frac{\varphi_i - \lambda}{L_i} - \alpha_1\right] = 0, \tag{A.11}$$

and α_1 is obtained by

$$\alpha_1 = \sum_{i=1}^{2} \frac{\varphi_i - \lambda}{L_i}. \tag{A.12}$$

Using Eq.(A.10), Eq.(A.12), $\lambda = \varphi_3 + \chi_1$, χ_i and $\pi = 0$, we finally have

$$\mathcal{H}'_{\rm em} = \sum_{i=1}^{2} \frac{(\varphi_i - \varphi_3)^2}{2L_i} + \sum_{i=1}^{3} \left\{ \frac{Q_i^2}{2C_i} - Q_i(V_i - V_c) \right\}. \tag{A.13}$$

Q_i and φ_i can be quantized as

$$[Q_i, \varphi_j] = \mathrm{i}\hbar\delta_{i,j} \;\; , \;\; [Q_i, Q_j] = [\varphi_i, \varphi_j] = 0. \tag{A.14}$$

$\mathcal{H}'_{\rm em}$ can be diagonalized by the canonical transformation from (φ_i, Q_i) to (φ'_i, Q'_i) and is obtained by

$$\mathcal{H}_{\rm em} = \mathcal{H}_{\rm env} + \mathcal{H}_{\rm c} \;, \tag{A.15}$$

where

$$\mathcal{H}_{\rm env} = \sum_{j=1}^{2} \Big\{ \frac{(\omega_j/\omega_L)^2}{2L_\Sigma}\varphi_j'^2 + \frac{Q_j'^2}{2C} - Q'_j(V'_j - V'_c) \Big\}, \tag{A.16}$$

$$\mathcal{H}_{\rm c} = \Big(\frac{q}{e} - n_c\Big)^2 U, \tag{A.17}$$

$$U \equiv \frac{e^2}{2C_\Sigma}, \tag{A.18}$$

$$n_c \equiv -\sum_{j=1}^{3} \frac{C_j V_j}{e} + \frac{eV_c}{2U}, \tag{A.19}$$

where $\mathcal{H}_{\rm env}$ and $\mathcal{H}_{\rm c}$ are Hamiltonian of environment effect coupled to external circuit and charging energy of island. Furthermore, U and n_c are, respectively, charging energy in the island and charge off-set of C-SET. Note that n_c includes the chemical potential of the island and self-consistently determined by imposing current continuity condition. Here $L_\Sigma \equiv L_1 + L_2, C \equiv C_1C_2/(C_1 + C_2), C_\Sigma \equiv C_1 + C_2 + C_3$, and $\omega_L \equiv 1/\sqrt{L_\Sigma C}$. Two kinds of non-zero eigenmodes of the electromagnetic environment $\omega_\pm$ ($+$ for $j = 1$ and $-$ for $j = 2$) are defined as

$$\begin{aligned}\Big(\frac{\omega_\pm}{\omega_{\rm L}}\Big)^2 =& \frac{1}{2}\Big(\frac{\kappa_1 + \kappa_3}{l_1} + \frac{\kappa_2 + \kappa_3}{l_2}\Big) \\ &\pm \sqrt{\frac{1}{4}\Big(\frac{\kappa_1}{l_1} - \frac{\kappa_2}{l_2}\Big)^2 + \Big[\frac{\kappa_3}{2}\Big(\frac{1}{l_1} + \frac{1}{l_2}\Big)\Big]^2 + \frac{\kappa_3}{2}\Big(\frac{1}{l_1} - \frac{1}{l_2}\Big)\Big(\frac{\kappa_1}{l_1} - \frac{\kappa_2}{l_2}\Big)},\end{aligned} \tag{A.20}$$

where $\kappa_i = C/C_i$ $(i = 1, 2, 3)$ and $l_j = L_j/L_\Sigma$ ($j = 1, 2$). In Fig.A.2 $\omega_\pm$ are shown as function of ratio of external inductance L_1/L_2. The canonical variables Q'_i and φ'_i satisfy the following commutation relation:

$$[Q'_i, \varphi'_j] = \mathrm{i}\hbar\delta_{i,j} \;\; , \;\; [Q'_i, Q'_j] = [\varphi'_i, \varphi'_j] = 0, \tag{A.21}$$

where charge operators Q'_1 and Q'_2 are carried by external circuit, and its spectrum is continuous. On the other hand, Q'_3 is related to the charging of island q as follows:

$$Q'_3 = -\sqrt{\frac{C}{C_\Sigma}}q, \tag{A.22}$$

where q occurred by single electron tunneling is quantized by

$$q|m\rangle = me|m\rangle, \tag{A.23}$$

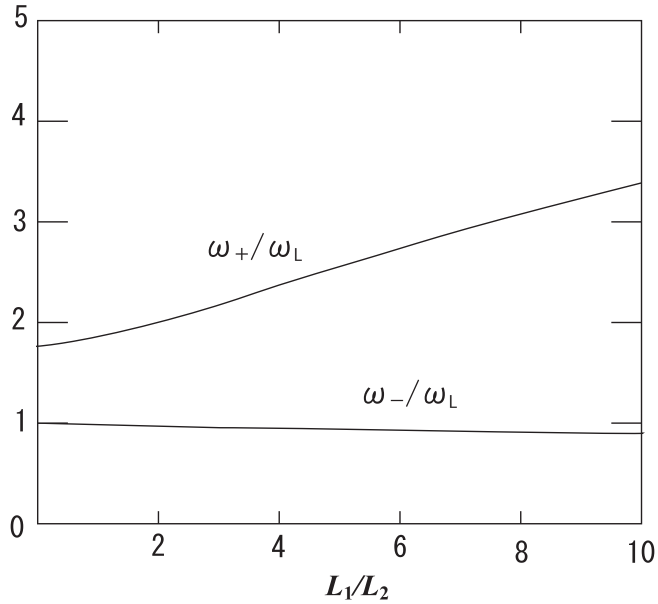

Fig. A.2 The non-zero eigenmodes of the electromagnetic environment $\omega_\pm$ versus environmental asymmetry. Parameters are tentatively chosen as $\omega_\pm/\omega_L$ vs L_1/L_2. $C_1/C_2 = 1, C_3/C_2 = 1$.

where m and $|m\rangle$ are an integer and an eigenstate of q when the island charge is me. Therefore a spectrum of Q'_3 is quantized. The tunnel Hamiltonian describing tunneling between double junctions as following:

$$H_\mathrm{T} = \sum_{i=1,2}\left\{\sum_{\boldsymbol{kk'}\sigma} T^{(i)}_{\boldsymbol{kk'}} \mathrm{e}^{\mathrm{i}\frac{e}{\hbar}\phi_i} a^{(i)\dagger}_{\boldsymbol{k}\sigma} a^{(c)}_{\boldsymbol{k'}\sigma} + T^{(i)*}_{\boldsymbol{kk'}} \mathrm{e}^{-\mathrm{i}\frac{e}{\hbar}\phi_i} a^{(c)\dagger}_{\boldsymbol{k'}\sigma} a^{(i)}_{\boldsymbol{k}\sigma}\right\}, \tag{A.24}$$

where

$$\phi_i(t) = V_i t + \varphi_i(t), \tag{A.25}$$

$$\varphi_i(t) = \sum_{j=1}^{3} \sqrt{\kappa_i}\eta_{ij}\varphi'_j = \varphi_{i,\mathrm{env}} + \varphi_{i,\mathrm{c}}, \tag{A.26}$$

$$\varphi_{i,\mathrm{env}} \equiv \sum_{j=1}^{2} \sqrt{\kappa_i}\eta_{ij}\varphi'_j\ ,\quad \varphi^{(\mathrm{c})}_i \equiv \sqrt{\kappa_i}\eta_{i3}\varphi'_3, \tag{A.27}$$

and η_{ij} is an ij component of Unitary matrix to transform to diagonal picture.

A.2 Correlation functions of phases

Both environmental and charging effects are specified by correlation functions of phases which are canonical conjugate to continuous charge and quantized charge. Phase correlation functions which describe in Keldysh space of the electromagnetic environmental effect $\check{\mathcal{F}}^{(i)}_{\lambda\lambda',\mathrm{env}}(t,t')$ and the charging effect $\check{\mathcal{F}}^{(i)}_{\lambda\lambda',\mathrm{c}}(t,t')$ are given by

$$\check{\mathcal{F}}^{(i)}_{\lambda\lambda',\mathrm{env}}(t,t') \equiv -\mathrm{i}\langle\, T_{C_K}\ \mathrm{e}^{\mathrm{i}\lambda\frac{e}{\hbar}\varphi^{(\alpha)}_{i,\mathrm{env}}(t)}\mathrm{e}^{\mathrm{i}\lambda'\frac{e}{\hbar}\varphi^{(\alpha)}_{i,\mathrm{env}}(t')}\rangle_{K_0}, \tag{A.28}$$

$$\check{\mathcal{F}}^{(i)}_{\lambda\lambda',\mathrm{c}}(t,t') \equiv -\mathrm{i}\langle T_{C_K}\mathrm{e}^{\mathrm{i}\lambda\frac{e}{\hbar}\varphi_{i,c}(t)}\mathrm{e}^{\mathrm{i}\lambda'\frac{e}{\hbar}\varphi_{i,c}(t')}\rangle_{K_0} = \check{\mathcal{F}}^{(i)}_{\lambda-\lambda,\mathrm{c}}(t-t')\delta_{\lambda',-\lambda}. \tag{A.29}$$

Environmental effect Hamiltonian $H_{\rm env}$ is a type of harmonic oscillator, so introducing creation and annihilation operator of boson $b_j^\dagger$ and b_j, $H_{\rm env}$ is enable to be rewritten by

$$H_{\rm env} = \sum_\alpha \sum_{j=1}^{2} \hbar\omega_j \left(b_j^\dagger b_j + \frac{1}{2} \right) , \tag{A.30}$$

where creation and annihilation operators are given by

$$\begin{aligned} b_j^\dagger &= \sqrt{\frac{\omega_j C}{2\hbar}}\varphi' - {\rm i}\frac{1}{\sqrt{2\hbar\omega_j C}}\big(Q' - CV_j'\big), \\ b_j &= \sqrt{\frac{\omega_j C}{2\hbar}}\varphi' + {\rm i}\frac{1}{\sqrt{2\hbar\omega_j C}}\big(Q' - CV_j'\big), \end{aligned} \tag{A.31}$$

with $[b_j, b_{j'}^\dagger] = \delta_{jj'}$ and using Campbell-Baker-Hausdorff formula

$$\mathrm{e}^{A+B} = \mathrm{e}^{A}\mathrm{e}^{B}\mathrm{e}^{-[A,B]/2}, \tag{A.32}$$

only if $[A,[A,B]] = [B,[A,B]] = 0$, environmental phase correlation function in Eq.(A.28) is obtained as

$$\begin{aligned} \mathcal{F}^{\mu\mu'(i)}_{+-,\rm env}(t,t') &= \exp\left\{ \sum_{j=1}^{2} \frac{\kappa_i \eta_{ij}^2 E_c}{\hbar\omega_j} J^{(i)\mu\mu'}_{+-}(t,t') \right\} \\ &= \mathcal{F}^{\mu\mu'(i)}_{-+,\rm env}(t,t') , \end{aligned} \tag{A.33}$$

where $\mu\mu'$ means the matrix components in Keldysh space and the diagonal components are given by

$$\begin{pmatrix} J^{(i)--}_{+-}(t,t') \\ J^{(i)++}_{+-}(t,t') \end{pmatrix} = \coth\frac{\beta\hbar\omega_j}{2}\big(\cos\omega_j(t-t') - 1\big) \mp {\rm i}\sin\omega_j|t-t'|, \tag{A.34}$$

and the off-diagonal components are given by

$$\begin{pmatrix} J^{(i)-+}_{+-,\rm c}(t,t') \\ J^{(i)+-}_{+-,\rm c}(t,t') \end{pmatrix} = \coth\frac{\beta\hbar\omega_j}{2}\big(\cos\omega_j(t-t') - 1\big) \pm {\rm i}\sin\omega_j(t-t'), \tag{A.35}$$

and charging energy is given by $E_c = e^2/2C$. Note that $\check{\mathcal{F}}^{(i)}_{\lambda\lambda',\rm env}(t,t')$ depends on environmental impedance.

On the other hand, correlation function $\check{\mathcal{F}}^{(i)}_{\lambda\lambda',\rm c}(t,t')$ is independent of environmental impedance. In this case calculation is also straightforward to note that

$$\mathrm{e}^{\pm{\rm i}\frac{e}{\hbar}\varphi_{i,c}}|m\rangle = |m \pm 1\rangle, \tag{A.36}$$

for the eigenstate of $q = -\sqrt{C_\Sigma/C}\cdot Q_3'$, we obtain

$$\begin{aligned} \begin{pmatrix} \mathcal{F}^{(i)--}_{-+,\rm c}(t,t') \\ \mathcal{F}^{(i)++}_{-+,\rm c}(t,t') \end{pmatrix} &= -{\rm i}\frac{\displaystyle\sum_{m=0}^{\infty} \mathrm{e}^{-\beta U(m-n_c)^2}\mathrm{e}^{-{\rm i}\frac{U}{\hbar}[2(m-n_c)(t-t')\pm|t-t'|]}}{\displaystyle\sum_{m=0}^{\infty}\mathrm{e}^{-\beta U(m-n_c)^2}} \\ &= \begin{pmatrix} \mathcal{F}^{(i)++}_{+-,\rm c}(t,t') \\ \mathcal{F}^{(i)--}_{+-,\rm c}(t,t') \end{pmatrix}, \end{aligned} \tag{A.37}$$

$$\begin{pmatrix} \mathcal{F}^{(i)-+}_{-+,\mathrm{c}}(t,t') \\ \mathcal{F}^{(i)+-}_{-+,\mathrm{c}}(t,t') \end{pmatrix} = -\mathrm{i}\frac{\displaystyle\sum_{m=0}^{\infty} \mathrm{e}^{-\beta U(m-n_\mathrm{c})^2} \mathrm{e}^{-\mathrm{i}\frac{U}{\hbar}[2(m-n_\mathrm{c})\mp 1](t-t')}}{\displaystyle\sum_{m=0}^{\infty} \mathrm{e}^{-\beta U(m-n_\mathrm{c})^2}}$$

$$= \begin{pmatrix} \mathcal{F}^{(i)+-}_{+-,\mathrm{c}}(t,t') \\ \mathcal{F}^{(i)-+}_{+-,\mathrm{c}}(t,t') \end{pmatrix}. \tag{A.38}$$

Appendix B

Derivation of the cumulant generating function

The first derivative of CGF in Eq.(3.52) is

$$\frac{\partial\Omega(\{\chi_\alpha\})}{\partial(\mathrm{i}\chi_\alpha)} = \lim_{T_0\to\infty}\frac{1}{T_0}\frac{\partial_{\mathrm{i}\chi_\alpha}Z(\{\chi_\alpha\},T_0)}{Z(\{\chi_\alpha\},T_0)}, \tag{B.1}$$

where $Z(\{\chi_\alpha\},T_0)$ is the characteristic function defined in Eq.(3.51).

$$\begin{aligned}
\frac{\partial\Omega(\{\chi_\alpha\})}{\partial(\mathrm{i}\chi_\alpha)} &= \lim_{T_0\to\infty}\frac{1}{T_0}\frac{\partial_{\mathrm{i}\chi_\alpha}Z(\{\chi_\alpha\},\frac{T_0}{2},-\frac{T_0}{2})}{Z(\{\chi_\alpha\};\frac{T_0}{2},-\frac{T_0}{2})}\\
&= \lim_{T_0\to\infty}\frac{1}{T_0}\frac{1}{Z(\{\chi_\alpha\})}\Bigg\{\mathrm{Tr}\Bigg[\rho_0\,\frac{\partial U_+\Big(\{\chi_\alpha\};-\frac{T_0}{2},\frac{T_0}{2}\Big)}{\partial(\mathrm{i}\chi_\alpha)}U_-\Big(\{\chi_\alpha\};\frac{T_0}{2},-\frac{T_0}{2}\Big)\\
&\qquad + U_+\Big(\{\chi_\alpha\};-\frac{T_0}{2},\frac{T_0}{2}\Big)\frac{\partial U_-\Big(\{\chi_\alpha\};\frac{T_0}{2},-\frac{T_0}{2}\Big)}{\partial(\mathrm{i}\chi_\alpha)}\Bigg]\Bigg\}\\
&= \lim_{T_0\to\infty}\frac{1}{T_0}\frac{1}{\hbar}\frac{1}{Z(\{\chi_\alpha\},T_0)}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}}\mathrm{d}t_1\\
&\quad\times\Bigg\{\mathrm{Tr}\Big[\rho_0\,\{U_+(\{\chi_\alpha\};-\frac{T_0}{2},t_1)\Big(\partial_{\mathrm{i}\chi_\alpha}\hat{H}_{\mathrm{T},+}(\{\chi_\alpha\},t_1)\Big)\\
&\qquad\times U_+(\{\chi_\alpha\};t_1,\frac{T_0}{2})U_-(\{\chi_\alpha\};\frac{T_0}{2},-\frac{T_0}{2})\\
&\quad - U_+(\{\chi_\alpha\};\frac{T_0}{2},-\frac{T_0}{2})U_-(\{\chi_\alpha\};\frac{T_0}{2},t_1)\\
&\quad\times\Big(\partial_{\mathrm{i}\chi_\alpha}\hat{H}_{\mathrm{T},-}(\{\chi_\alpha\},t_1)\Big)U_-(\{\chi_\alpha\};t_1,-\frac{T_0}{2})\}\Big]\Bigg\}\\
&= \lim_{T_0\to\infty}\frac{1}{T_0}\frac{\mathrm{i}}{\hbar}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}}\mathrm{d}t_1\Big\{\langle\Big\langle\partial_{\mathrm{i}\chi_\alpha}\hat{H}_{\mathrm{T},+}(\{\chi_\alpha\},t_1)\Big)\Big\rangle_{K_0T_0}\\
&\qquad - \Big\langle\Big(\partial_{\mathrm{i}\chi_\alpha}\hat{H}_{\mathrm{T},-}(\{\chi_\alpha\},t_1)\Big)\Big\rangle_{K_0T_0}\Big\}\,.
\end{aligned} \tag{B.2}$$

Let us here define the counting field dependent Green's function with finite time interval $(-T_0/2, T_0/2)$ by

$$\check{\mathcal{G}}_{\nu\nu'}(\{\chi_\alpha\};\boldsymbol{k}\sigma,t;\boldsymbol{k}'\sigma',t') \equiv -\mathrm{i}\left\langle T_C \hat{\Psi}^{(\nu)}_{\boldsymbol{k}\sigma}(t)\hat{\Psi}^{(\nu')\dagger}_{\boldsymbol{k}'\sigma'}(t')\right\rangle_{K(\{\chi_\alpha\}),T_0}, \quad (\nu,\nu'=c,d_\alpha,a_\alpha) \tag{B.3}$$

$$\check{\mathcal{G}}^{(0)}_{d_\alpha i}(\boldsymbol{k}\sigma;t,t') \equiv -\mathrm{i}\left\langle T_C \hat{\Psi}^{(d_\alpha i)}_{\boldsymbol{k}\sigma}(t)\hat{\Psi}^{(d_\alpha i)\dagger}_{\boldsymbol{k}\sigma}(t')\right\rangle_{K_0 T_0}. \quad (i=1,2) \tag{B.4}$$

Note that the average $\langle\cdots\rangle_{T_0}$ is not genuine statistical average and these Green's functions do not depend time difference $t-t'$ in general, unless the limit $T_0\to\infty$ is taken. In what follows we do not express spin indices for simplicity, since the fundamental tunneling processes never change spin orientation.

Noting Eq.(3.56) and Eq.(3.57)

$$\check{T}_{cd_\alpha}(\chi_\alpha) \equiv \begin{pmatrix} \hat{T}_{cd_\alpha}\mathrm{e}^{\frac{\mathrm{i}}{2}\chi_\alpha\hat{\tau}_3}, & 0 \\ 0, & -\hat{T}_{cd_\alpha}\mathrm{e}^{-\frac{\mathrm{i}}{2}\chi_\alpha\hat{\tau}_3} \end{pmatrix} \equiv \begin{pmatrix} \check{T}^{--}_{cd_\alpha}(\chi_\alpha) & \check{T}^{-+}_{cd_\alpha}(\chi_\alpha) \\ \check{T}^{+-}_{cd_\alpha}(\chi_\alpha) & \check{T}^{++}_{cd_\alpha}(\chi_\alpha) \end{pmatrix} = \check{T}_{d_\alpha c}(\chi_\alpha)^* , \tag{B.5}$$

$$\check{T}_{a_\alpha d_\alpha}(\chi_\alpha) \equiv \begin{pmatrix} \hat{T}_{a_\alpha d_\alpha}\mathrm{e}^{\frac{\mathrm{i}}{2}\chi_\alpha\hat{\tau}_3}, & 0 \\ 0, & -\hat{T}_{a_\alpha d_\alpha}\mathrm{e}^{-\frac{\mathrm{i}}{2}\chi_\alpha\hat{\tau}_3} \end{pmatrix} \equiv \begin{pmatrix} \check{T}^{--}_{a_\alpha d_\alpha}(\chi_\alpha) & \check{T}^{-+}_{a_\alpha d_\alpha}(\chi_\alpha) \\ \check{T}^{+-}_{a_\alpha d_\alpha}(\chi_\alpha) & \check{T}^{++}_{a_\alpha d_\alpha}(\chi_\alpha) \end{pmatrix} = \check{T}_{d_\alpha a_\alpha}(\chi_\alpha)^* , \tag{B.6}$$

we obtain

$$\begin{aligned}\frac{\partial\Omega(\{\chi_\alpha\})}{\partial(\mathrm{i}\chi_\alpha)} = \lim_{T_0\to\infty}\frac{1}{T_0}\Big\{\frac{1}{4\hbar}\sum_{\boldsymbol{k}\boldsymbol{k}'}\int_{-T_0/2}^{T_0/2}\mathrm{Tr}_{NG} \\ \times\Big[&\check{T}^{--}_{cd_\alpha}(\chi_\alpha)\hat{\tau}_3\check{\mathcal{G}}^{--}_{d_\alpha c}(\boldsymbol{k},\boldsymbol{k}';t_-,t_-+\delta) - \check{T}^{--}_{d_\alpha c}(\chi_\alpha)\hat{\tau}_3\check{\mathcal{G}}^{--}_{cd_\alpha}(\boldsymbol{k},\boldsymbol{k}';t_-,t_-+\delta) \\ &+\check{T}^{++}_{cd_\alpha}(\chi_\alpha)\hat{\tau}_3\check{\mathcal{G}}^{++}_{d_\alpha c}(\boldsymbol{k},\boldsymbol{k}';t_++\delta,t_+) - \check{T}^{++}_{d_\alpha c}(\chi_\alpha)\hat{\tau}_3\check{\mathcal{G}}^{++}_{cd_\alpha}(\boldsymbol{k},\boldsymbol{k}';t_++\delta,t_+) \\ &+\check{T}^{--}_{a_\alpha d_\alpha}(\chi_\alpha)\hat{\tau}_3\check{\mathcal{G}}^{--}_{d_\alpha a_\alpha}(\boldsymbol{k},\boldsymbol{k}';t_-,t_-+\delta) - \check{T}^{--}_{d_\alpha a_\alpha}(\chi_\alpha)\hat{\tau}_3\check{\mathcal{G}}^{--}_{a_\alpha d_\alpha}(\boldsymbol{k},\boldsymbol{k}';t_-,t_-+\delta) \\ &+\check{T}^{++}_{a_\alpha d_\alpha}(\chi_\alpha)\hat{\tau}_3\check{\mathcal{G}}^{++}_{d_\alpha a_\alpha}(\boldsymbol{k},\boldsymbol{k}';t_++\delta,t_+) - \check{T}^{++}_{d_\alpha a_\alpha}(\chi_\alpha)\hat{\tau}_3\check{\mathcal{G}}^{++}_{a_\alpha d_\alpha}(\boldsymbol{k},\boldsymbol{k}';t_++\delta,t_+)\Big]\Big\} .\end{aligned} \tag{B.7}$$

Solving the equations of motion for $\check{\mathcal{G}}_{\nu\nu'}(\{\chi_\alpha\};\boldsymbol{k}\sigma,t_\lambda;\boldsymbol{k}'\sigma',t'_{\lambda'})$, we obtain

$$\check{\mathcal{G}}_{cd_\alpha}(\boldsymbol{k},\boldsymbol{k}';t,t') \simeq \frac{1}{\hbar}\sum_{\boldsymbol{k}''}\int_C \mathrm{d}\tau\ \check{\mathcal{G}}_{cc}(\boldsymbol{k},\boldsymbol{k}'';t,\tau)\check{T}_{cd_\alpha}(\chi_\alpha(\tau))\check{\mathcal{G}}^{(0)}_{d_\alpha 1}(\boldsymbol{k}';\tau,t') , \tag{B.8}$$

$$\check{\mathcal{G}}_{d_\alpha c}(\boldsymbol{k},\boldsymbol{k}';t,t') \simeq \frac{1}{\hbar}\sum_{\boldsymbol{k}''}\int_C \mathrm{d}\tau\ \check{\mathcal{G}}^{(0)}_{d_\alpha 1}(\boldsymbol{k};t,\tau)\check{T}_{d_\alpha c}(\chi_\alpha(\tau))\check{\mathcal{G}}_{cc}(\boldsymbol{k}'',\boldsymbol{k}';\tau,t') , \tag{B.9}$$

$$\check{\mathcal{G}}_{a_\alpha d_\alpha}(\boldsymbol{k},\boldsymbol{k}';t,t') \simeq \frac{1}{\hbar}\sum_{\boldsymbol{k}''}\int_C \mathrm{d}\tau\ \check{\mathcal{G}}_{cc}(\boldsymbol{k},\boldsymbol{k}'';t,\tau)\check{T}_{a_\alpha d_\alpha}(\chi_\alpha(\tau))\check{\mathcal{G}}^{(0)}_{d_\alpha 2}(\boldsymbol{k}';\tau,t') , \tag{B.10}$$

$$\check{\mathcal{G}}_{d_\alpha a_\alpha}(\boldsymbol{k},\boldsymbol{k}';t,t') \simeq \frac{1}{\hbar}\sum_{\boldsymbol{k}''}\int_C \mathrm{d}\tau\ \check{\mathcal{G}}^{(0)}_{d_\alpha 2}(\boldsymbol{k};t,\tau)\check{T}_{d_\alpha a_\alpha}(\chi_\alpha(\tau))\check{\mathcal{G}}_{a_\alpha a_\alpha}(\boldsymbol{k}'',\boldsymbol{k}';\tau,t') . \tag{B.11}$$

In deriving from Eq.(B.8) to Eq.(B.11) we only retained up to the two-body correlation for environmental and charging phases. Concerning the electromagnetic environment effect, this approximation never spoils a reasonable description of physics. Charge (voltage) fluctuation in low environmental impedance case and stabilization of Coulomb blockade (enhancement of Coulomb gap region etc.) in high impedance case are still well described. Furthermore, we also introduced approximation that the lowest order Green's functions of island electrons only convey informations of environmental and charging effects. Concerning the charging effect, it is known that Coulomb blockade related phenomena such as Coulomb staircase and Coulomb oscillation can be well described within this approximation. Note that the quenching of a certain class of tunneling processes due to charge quantization can be automatically excluded from the beginning, since the charging phase correlation is decoupled from the off-diagonal Gor'kov Green's functions.

Substituting these Dyson equations into Eq.(B.7) and noting that the lowest order island green's function is diagonal in Nambu-Gor'kov space, we arrive at

$$\begin{aligned}\frac{\partial\Omega(\{\chi_\alpha\})}{\partial(\mathrm{i}\chi_\alpha)} &= \lim_{T_0\to\infty}\frac{1}{T_0}\Big\{\frac{1}{4\hbar}\sum_{\boldsymbol{k}\boldsymbol{k}'}\int_C \mathrm{d}\tau\int_C \mathrm{d}\tau'\mathrm{Tr}_{NG\otimes SK} \\ &\times\Big[\check{T}_{cd_\alpha}(\chi_\alpha(\tau))\check{T}_{d_\alpha c}(\chi_\alpha(\tau'))\hat{\tau}_3\check{\mathcal{G}}_{cc}(\tau,\tau')\check{\mathcal{G}}^{(0)}_{d_\alpha 1}(\tau',\tau) \\ &\quad \check{T}_{a_\alpha d_\alpha}(\chi_\alpha(\tau))\check{T}_{d_\alpha a_\alpha}(\chi_\alpha(\tau'))\hat{\tau}_3\check{\mathcal{G}}_{a_\alpha a_\alpha}(\tau,\tau')\check{\mathcal{G}}^{(0)}_{d_\alpha 2}(\tau',\tau)\Big]\ . \end{aligned} \tag{B.12}$$

Furthermore, from the solution of the equation of motion for $\check{\mathcal{G}}_{cc}(t,t')$, we have

$$\begin{aligned}\check{\mathcal{G}}_{cc}(t,t') &= \check{\mathcal{G}}^{(0)}_{c}(t,t') + \int_C\int_C \mathrm{d}\tau\mathrm{d}\tau'\check{\mathcal{G}}^{(0)}_{c}(t,\tau) \\ &\times\left[\sum_\alpha \check{T}_{cd_\alpha}(\chi_\alpha(\tau))\check{T}_{d_\alpha c}(\chi_\alpha(\tau'))\check{\mathcal{G}}^{(0)}_{d_\alpha 1}(\tau,\tau')\right]\check{\mathcal{G}}_{cc}(\tau',t')\ , \\ &\equiv \check{\mathcal{G}}^{(0)}_{c}(t,t') + \int_C\int_C \mathrm{d}\tau\mathrm{d}\tau'\check{\mathcal{G}}^{(0)}_{c}(t,\tau)\frac{1}{\hbar}\check{\mathcal{S}}_c(\chi_\alpha(\tau),\chi_\alpha(\tau');\tau,\tau')\ \check{\mathcal{G}}_{cc}(\tau',t')\ ,\end{aligned} \tag{B.13}$$

$$\begin{aligned}\check{\mathcal{G}}_{a_\alpha a_\alpha}(t,t') &= \check{\mathcal{G}}^{(0)}_{a_\alpha}(t,t') + \int_C\int_C \mathrm{d}\tau\mathrm{d}\tau'\check{\mathcal{G}}^{(0)}_{a_\alpha}(t,\tau) \\ &\times\left[\check{T}_{cd_\alpha}(\chi_\alpha(\tau))\check{T}_{d_\alpha c}(\chi_\alpha(\tau'))\check{\mathcal{G}}^{(0)}_{d_\alpha 1}(\tau,\tau')\right]\check{\mathcal{G}}_{a_\alpha a_\alpha}(\tau',t')\ , \\ &\equiv \check{\mathcal{G}}^{(0)}_{a_\alpha}(t,t') + \int_C\int_C \mathrm{d}\tau\mathrm{d}\tau'\frac{1}{\hbar}\check{\mathcal{S}}_\alpha(\chi_\alpha(\tau),\chi_\alpha(\tau');\tau,\tau')\ \check{\mathcal{G}}_{a_\alpha a_\alpha}(\tau',t')\ ,\end{aligned} \tag{B.14}$$

where we defined *self-energies* $\check{\mathcal{S}}_c$ and $\check{\mathcal{S}}_\alpha$ for $\check{\mathcal{G}}_{cc}$ and $\check{\mathcal{G}}_{a_\alpha a_\alpha}$, respectively.

Now let us take the limit $T_0\to\infty$. Noting that $\check{\mathcal{G}}(t,t')\to\check{G}(t-t')$ and $\check{\mathcal{S}}(t,t')\to\check{\Sigma}(t-t')$, and performing the Fourier transformation, we arrive at

$$\begin{aligned}\frac{\partial\Omega(\{\chi_\alpha\})}{\partial(\mathrm{i}\chi_\alpha)} =& \frac{1}{2}\sum_\sigma\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi} \\ &\times\mathrm{Tr}_{SK\otimes NG}\Big[-\frac{\check{g}_c(\omega)(\partial_{\mathrm{i}\chi_\alpha}\check{\Sigma}_c(\{\chi_\alpha\},\omega)/\hbar)}{\check{1}-\check{g}_{cc}(\omega)\check{\Sigma}_c(\{\chi_\alpha\},\omega)/\hbar} - \frac{\check{g}_{a_\alpha}(\omega)(\partial_{\mathrm{i}\chi_\alpha}\check{\Sigma}_{a_\alpha}(\chi_\alpha,\omega)/\hbar)}{\check{1}-\check{g}_{a_\alpha}(\omega)\check{\Sigma}_{a_\alpha}(\chi_\alpha,\omega)/\hbar}\Big]\ .\end{aligned} \tag{B.15}$$

Therefore, we finally have the following expression for cumulant generation function:

$$\Omega(\{\chi_\alpha\}) = \frac{1}{2}\sum_\sigma \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \mathrm{Tr}_{SK\otimes NG} \left\{ \log\left[\frac{\check{1} - \check{g}_{cc}(\omega)\check{\Sigma}_c(\{\chi_\alpha\},\omega)/\hbar}{\check{1} - \check{g}_{cc}(\omega)\check{\Sigma}_c(0;\omega)/\hbar}\right] \right.$$
$$\left. + \sum_{\alpha=\mathrm{L,R}} \log\left[\frac{\check{1} - \check{g}_{a_\alpha a_\alpha}(\omega)\check{\Sigma}_{a_\alpha}(\chi_\alpha,\omega)/\hbar}{\check{1} - \check{g}_{a_\alpha a_\alpha}(\omega)\check{\Sigma}_{a_\alpha}(0;\omega)/\hbar}\right] \right\} . \tag{B.16}$$

Note that the result satisfies boundary condition

$$\Omega(\{\chi_\alpha\} = 0) = 0 \, . \tag{B.17}$$

Appendix C

The lowest order Green's functions

The lowest order Green's functions of electrons in various electrodes are given as follows :

(i) Normalconducting Electrodes

$$g^{-+}_{a_\alpha 11}(\omega) = g^{-+}_{a_\alpha 22}(\omega) = 2\mathrm{i}\pi\hbar N_{a_\alpha}(0)\ n_{\mathrm{F}}(\epsilon)\ , \tag{C.1}$$

$$g^{+-}_{a_\alpha 11}(\omega) = g^{+-}_{a_\alpha 22}(\omega) = -2\mathrm{i}\pi\hbar N_{a_\alpha}(0)\ n_{\mathrm{F}}(-\epsilon)\ , \tag{C.2}$$

$$g^{\mathrm{R}}_{a_\alpha 11}(\omega) = g^{\mathrm{R}}_{a_\alpha 22}(\omega) = \left[g^{\mathrm{A}}_{a_\alpha 11}(\omega)\right]^* = \left[g^{\mathrm{A}}_{a_\alpha 22}(\omega)\right]^* = -\mathrm{i}\pi\hbar N_{a_\alpha}(0)\ . \tag{C.3}$$

(ii) Superconducting Electrodes

$$g^{-+}_{c11}(\omega) = g^{-+}_{c22}(\omega) = 2\mathrm{i}\pi\hbar N_c(0)\frac{|\epsilon|}{\sqrt{\epsilon^2-\Delta^2}}\ n_{\mathrm{F}}(\epsilon) \qquad (|\epsilon| > \Delta)\ , \tag{C.4}$$

$$g^{+-}_{c11}(\omega) = g^{+-}_{c22}(\omega) = -2\mathrm{i}\pi\hbar N_c(0)\frac{|\epsilon|}{\sqrt{\epsilon^2-\Delta^2}}\ n_{\mathrm{F}}(-\epsilon) \quad (|\epsilon| > \Delta)\ , \tag{C.5}$$

$$\begin{aligned} g^{\mathrm{R}}_{c11}(\omega) &= g^{\mathrm{R}}_{c22}(\omega) = \left[g^{\mathrm{A}}_{c11}(\omega)\right]^* = \left[g^{\mathrm{A}}_{c22}(\omega)\right]^* \\ &= \begin{cases} -\pi\hbar N_c(0)\dfrac{\epsilon}{\sqrt{\Delta^2-\epsilon^2}} & (|\epsilon| < \Delta) \\ -\mathrm{i}\pi\hbar N_c(0)\dfrac{|\epsilon|}{\sqrt{\epsilon^2-\Delta^2}} & (|\epsilon| > \Delta) \end{cases}\ , \end{aligned} \tag{C.6}$$

$$g^{-+}_{c12}(\omega) = g^{-+}_{c21}(\omega) = 2\mathrm{i}\cdot\mathrm{sgn}(\epsilon)\pi\hbar N_c(0)\frac{\Delta}{\sqrt{\epsilon^2-\Delta^2}}\ n_{\mathrm{F}}(\epsilon) \qquad (|\epsilon| > \Delta)\ , \tag{C.7}$$

$$g^{+-}_{c12}(\omega) = g^{+-}_{c21}(\omega) = -2\mathrm{i}\cdot\mathrm{sgn}(\epsilon)\pi\hbar N_c(0)\frac{\Delta}{\sqrt{\epsilon^2-\Delta^2}}\ n_{\mathrm{F}}(-\epsilon) \quad (|\epsilon| > \Delta)\ , \tag{C.8}$$

$$\begin{aligned} g^{\mathrm{R}}_{c12}(\omega) &= g^{\mathrm{R}}_{c21}(\omega) = \left[g^{\mathrm{A}}_{c21}(\omega)\right]^* = \left[g^{\mathrm{A}}_{c12}(\omega)\right]^* \\ &= \begin{cases} -\pi\hbar N_c(0)\dfrac{\Delta}{\sqrt{\Delta^2-\epsilon^2}} & (|\epsilon| < \Delta) \\ -\mathrm{i}\cdot\mathrm{sgn}(\epsilon)\pi\hbar\ N_c(0)\dfrac{\Delta}{\sqrt{\epsilon^2-\Delta^2}} & (|\epsilon| > \Delta) \end{cases}\ . \end{aligned} \tag{C.9}$$

(iii) Normalconducting Islands

$$g^{-+}_{d_{\alpha i}11}(\omega) = 2\mathrm{i}\pi\hbar N_{d_\alpha}(0)\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega'}{2\pi}\;\mathrm{i}\mathcal{F}^{(\alpha i)-+}_{+-}(\omega')n_{\mathrm{F}}(\epsilon-\epsilon')\;, \tag{C.10}$$

$$g^{+-}_{d_{\alpha i}11}(\omega) = -2\mathrm{i}\pi\hbar N_{d_\alpha}(0)\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega'}{2\pi}\;\mathrm{i}\mathcal{F}^{(\alpha i)+-}_{+-}(\omega')\big[1-\;n_{\mathrm{F}}(\epsilon-\epsilon')\big]\;, \tag{C.11}$$

$$\begin{aligned} g^{\mathrm{R}}_{d_{\alpha i}11}(\omega) &= \big[g^{\mathrm{A}}_{d_\alpha 11}(\omega)\big]^* \\ &= -\mathrm{i}\pi\hbar N_{d_\alpha}(0)\left\{1+\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega'}{2\pi}\mathrm{i}\mathcal{F}^{(\alpha i)\mathrm{R}}_{+-}(\omega')\Big[1-2n_{\mathrm{F}}(\epsilon-\epsilon')\Big]\right\}\;, \end{aligned} \tag{C.12}$$

$$g^{-+}_{d_{\alpha i}22}(\omega) = 2\mathrm{i}\pi\hbar N_{d_\alpha}(0)\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega'}{2\pi}\;\mathrm{i}\mathcal{F}^{(\alpha i)-+}_{-+}(\omega')\;n_{\mathrm{F}}(\epsilon-\epsilon')\;, \tag{C.13}$$

$$g^{+-}_{d_{\alpha i}22}(\omega) = -2\mathrm{i}\pi\hbar N_{d_\alpha}(0)\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega'}{2\pi}\;\mathrm{i}\mathcal{F}^{(\alpha i)+-}_{-+}(\omega')\big[1-n_{\mathrm{F}}(\epsilon-\epsilon')\big]\;, \tag{C.14}$$

$$\begin{aligned} g^{\mathrm{R}}_{d_{\alpha i}22}(\omega) &= \big[g^{\mathrm{A}}_{d_{\alpha i}22}(\omega)\big]^* \\ &= -\mathrm{i}\pi\hbar N_{d_\alpha}(0)\left\{1+\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega'}{2\pi}\mathrm{i}\mathcal{F}^{(\alpha i)\mathrm{R}}_{-+}(\omega')\big[1-2n_{\mathrm{F}}(\epsilon-\epsilon')\big]\right\}, \end{aligned} \tag{C.15}$$

where $\mathcal{F}^{(\alpha i)}_{\lambda\lambda'}(\omega) = \mathcal{F}^{(\alpha i)}_{\lambda\lambda',\mathrm{env}}(\omega)\mathcal{F}^{(\alpha i)}_{\lambda\lambda',\mathrm{c}}(\omega)$. In the above, $\epsilon = \hbar\omega$, and $n_{\mathrm{F}}(\epsilon) = [\exp(\beta\epsilon)+1]^{-1}$ and $N_\nu(0)$ are, respectively, Fermi distribution function and the density of states at Fermi energy for the metal of electrode ν.

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